Name: ______________________________ ________ / 12

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have **30** minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- $\boxed{\text{Do } \textbf{not} \text{ simplify your expressions}}$.
- **Your final answers should start with** $f'(x) =$, $\frac{dy}{dx} =$ or something similar.
- $\boxed{\text{Box}}$ your final answer.

Note that this sample derivative proficiency is slightly different from the actual derivative proficiency, because there are a few functions ($\ln(x)$, *inverse trig functions, implicit differentiation in general) that we haven't covered yet.*

1. **[12 points]** Compute the derivatives of the following functions.

a. $f(x) = \sqrt[5]{x} + 4x^3 + \dfrac{x-\sqrt{2}}{9}$ $= x^{1/5} + 4x^3 + \frac{x}{9} - \frac{\sqrt{2}}{9}$

$f'(x) = \frac{1}{5}x^{-4/5} + 4(3x^2) + \frac{1}{9}$

b. $y = x^3\tan(x)$

$y' = x^3\sec^2(x) + \tan(x)(3x^2)$

c. $y = \dfrac{\sec(x)}{1+e^x}$

$$y' = \frac{(1+e^x)[\sec(x)\tan(x)] - \sec(x)(e^x)}{(1+e^x)^2}$$

d. $y = \sin(ax)e^{bx^2}$ where a and b are fixed constants.

$$y' = \sin(ax)\left(e^{bx^2}(2bx)\right) + e^{bx^2}(\cos(ax))(a)$$

e. $f(x) = \dfrac{\cos(x)}{\sin(x)}$

$$f'(x) = \frac{\sin(x)(-\sin(x)) - (\cos(x))(\cos(x))}{(\sin(x))^2}$$

$$= \frac{-(\sin(x))^2 - (\cos(x))^2}{(\sin(x))^2}$$

$$= \frac{-(1)}{(\sin(x))^2} = -(\csc(x))^2$$

← You might have also noticed the function is $f(x) = \cot(x)$

f. $g(x) = \sqrt{2 + \sin^2(6x)} = \left(2 + (\sin(6x))^2\right)^{1/2}$

$$g'(x) = \frac{1}{2}\left(2 + (\sin(6x))^2\right)^{-1/2}(2\sin(6x))(\cos(6x))(6)$$

Notice $g(x)$ is the composition of <u>four</u> functions:

$f_1(\square) = \sqrt{\square}$

$f_2(\square) = 2 + \square^2$

$f_3(\square) = \sin(\square)$

$f_4(\square) = 6\square$

g. $y = \tan\left(x^3 \cdot 5^x\right)$

$$y' = \sec^2(x^3 \cdot 5^x)\left(x^3 \cdot (5^x \ln(5)) + (3x^2)(5^x) \right)$$

h. $f(z) = \sec(\sqrt{z})$ $= \sec(z^{1/2})$

$$f'(z) = \sec(z^{1/2})\tan(z^{1/2})\left(\frac{1}{2} z^{-1/2}\right)$$

i. $y = \sin\left(\dfrac{x}{x-3}\right)$

$$y' = \cos\left(\frac{x}{x-3}\right)\left[\frac{(x-3)(1) - x(1)}{(x-3)^2}\right]$$

j. $h(x) = \cos\left(e^{\pi x} - (4x)^9\right)$

$h'(x) = -\sin\left(e^{\pi x} - (4x)^9\right)\left(e^{\pi x}\cdot\pi - 9(4x)^8(4)\right)$

k. $g(x) = (\sin(x^2+x))^5$

$g'(x) = 5(\sin(x^2+x))^4 \cos(x^2+x)(2x+1)$

l. $f(x) = \frac{1}{9x} = \frac{1}{9}x^{-1}$

$f'(x) = \frac{1}{9}(-1x^{-2}) \qquad \left(= \frac{-1}{9x^2}\right)$