1. Find all critical points of the function $f(x) = \sin(x)^{1/3}$.

C.p: where f' undefined or $f' = 0$.

$$f'(x) = \frac{1}{3}(\sin(x))^{-2/3}(\cos x) = \frac{\cos x}{3(\sin(x))^{2/3}}$$

$f' = 0$ when $\cos x = 0$. So $x = \ldots, \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots = \frac{(2k+1)\pi}{2}$ for k integer

$= \pi k + \frac{\pi}{2}$ k an integer.

f' undefined when $\sin x = 0$. So $x = \ldots, -2\pi, -\pi, 0, \pi, 2\pi, \ldots = \pi k$, k integer.

ANSWER $f(x)$ has critical points when $x = \frac{\pi k}{2}$ for k integer.

2. Find the absolute maximum and minimum values of $f(x) = e^{-x^2}$ on the interval $[-2, 3]$, and the locations where those values are attained.

$f'(x) = -2xe^{-x^2}$

① Find critical points: f' undefined? Never.
$f' = 0$? when $x = 0$.

② Table of values:

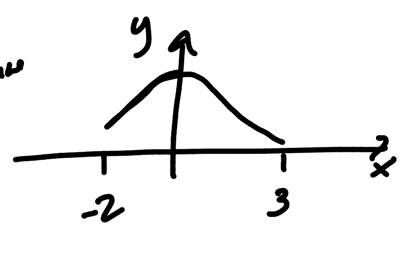
x	$y = e^{-x^2}$
-2	$y = \frac{1}{e^4}$
0	$y = 1$ ← largest
3	$y = \frac{1}{e^9}$ ← smallest

③ ANSWER:

1 is the abs. max at $x=0$

$\frac{1}{e^9}$ is the abs min at $x=3$

picture



3. A ball thrown in the air at time $t = 0$ has a height given by

$$h(t) = h_0 + v_0 t - \frac{1}{2} g_0 t^2$$

meters where t is measured in seconds, h_0 is the height at time 0, v_0 is the velocity (in meters per second) at time 0 and g_0 is the constant acceleration due to gravity (in m/s^2). Assuming $v_0 > 0$, find the time that the ball attains its maximum height. Then find the maximum height.

Goal: Maximize $h(t)$.

$$h'(t) = 0 + v_0 - \frac{1}{2} g_0 (2t) = v_0 - g_0 t$$

① Find c.points. h' never undefined

$h' = 0$ if $v_0 - g_0 t = 0$. So $t = v_0/g_0$

answer 1 : $h(t)$ attains its maximum height when $t = v_0/g_0$ seconds.

② Plug $t = v_0/g_0$ into $h(t)$ to find maximum height:

$$h(v_0/g_0) = h_0 + v_0\left(v_0/g_0\right) - \frac{1}{2} g_0 \left(\frac{v_0}{g_0}\right)^2$$

$$= h_0 + \frac{v_0^2}{g_0} - \frac{1}{2}\frac{v_0^2}{g_0} = h_0 + \frac{v_0^2}{2g_0} \text{ meters.}$$