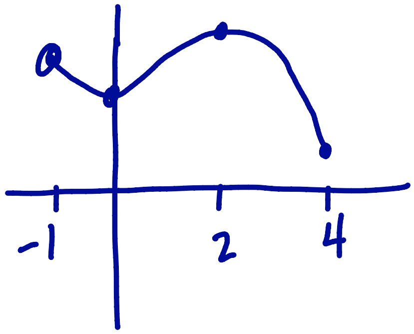
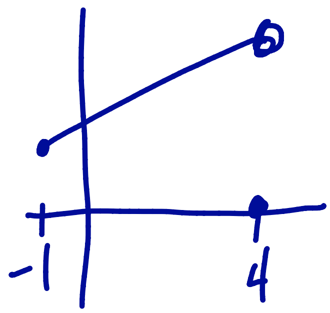
1. Sketch a graph $f(x)$ whose domain is the interval $[-1, 4]$ with the following properties:

(a) f is continuous on the interval $[-1, 4]$, has a local minimum at $x = 0$, an absolute minimum at $x = 4$ and an absolute maximum at $x = 2$.

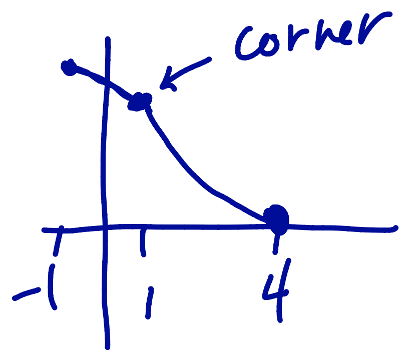


(b) f has a ~~local~~ abs. minimum but no absolute maximum



Would have to be discontinuous

(c) f has a critical point at $x = 1$ but ~~f has~~ no maximum or minimum (of any type) at $x = 1$.



2. Find the maximum and minimum values of $f(x) = x - x^{1/3}$ on the interval [-1,4]. Determine where those maximum and minimum values occur.

$$f'(x) = 1 - \frac{1}{3}x^{-2/3} = 1 - \frac{1}{3x^{2/3}}$$

① Find critical points

f' undefined where $x=0$

$f' = 0$ when $0 = 1 - \frac{1}{3x^{2/3}}$

$$\frac{1}{3x^{2/3}} = 1$$

$$\frac{1}{3} = x^{2/3}$$

$$x = \pm\left(\frac{1}{3}\right)^{3/2} = \pm\left(\frac{1}{3}\right)^{3/2} \approx \pm 0.19$$

$$\left(\frac{1}{3}\right)^3 = x^2$$

$$x = \pm\sqrt{\left(\frac{1}{3}\right)^3}$$

② Make a table of crit. pts + end points + plug into $f(x)$.

x	$y = x - x^{1/3}$
-1	$-1 - (-1)^{1/3} = 0$
0	$0 - 0 = 0$
$\left(\frac{1}{3}\right)^{3/2}$	$\left(\frac{1}{3}\right)^{3/2} - \left(\frac{1}{3}\right)^{1/2} \approx -0.38$
4	$4 - 4^{1/3} \approx 2.4$
$-\left(\frac{1}{3}\right)^{3/2}$	$-\left(\frac{1}{3}\right)^{3/2} - \left(-\left(\frac{1}{3}\right)^{\frac{1}{2}}\right) \approx 0.38$

③ Answer:

max. value is $y = 2.4$ at $x = 4$

min. value is $y = -0.38$ at $x = \left(\frac{1}{3}\right)^{3/2}$

* See sketch at end →

$f(x) = x + x^{-1}$

3. Find the maximum and minimum values of $f(x) = x + \dfrac{1}{x}$ on the interval [1/5,4]. Determine where those maximum and minimum values occur.

$f'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2}$

① Find crit. pts.

f' undefined at $x=0$ ← not in domain

$f' = 0$ when $1 - \frac{1}{x^2} = 0$

$x^2 = 1$

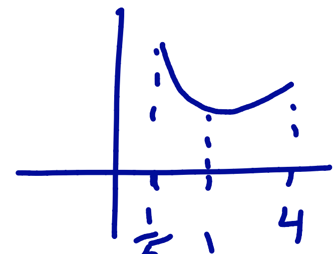
$x = \pm 1$ (but $x=-1$ not in domain)

② Make table

x	$y = x + \frac{1}{x}$
$\frac{1}{5}$	$\frac{1}{5} + 5 = 5.2$
$\cancel{0}$	
$+1$	$1+1=2$
4	$4 + \frac{1}{4} = 4.25$

③ ANSWER: max. value $y = 5.2$ at $x = 1/5$

min value $y = 2$ at $x = 1$



4. Find the maximum and minimum values of $f(x) = x^{2/3}$ on the interval [-8,8]. Determine where those maximum and minimum values occur.

$f'(x) = \frac{2}{3} x^{-1/3}$

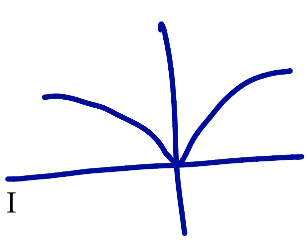
① Find crit. pts

f' undef. at $x=0$

$f' = 0$ when $0 = \frac{2}{3} x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$

But this has no solution!

So f' is never zero.



② table

x	$y = x^{2/3}$
-8	4
0	0
8	4

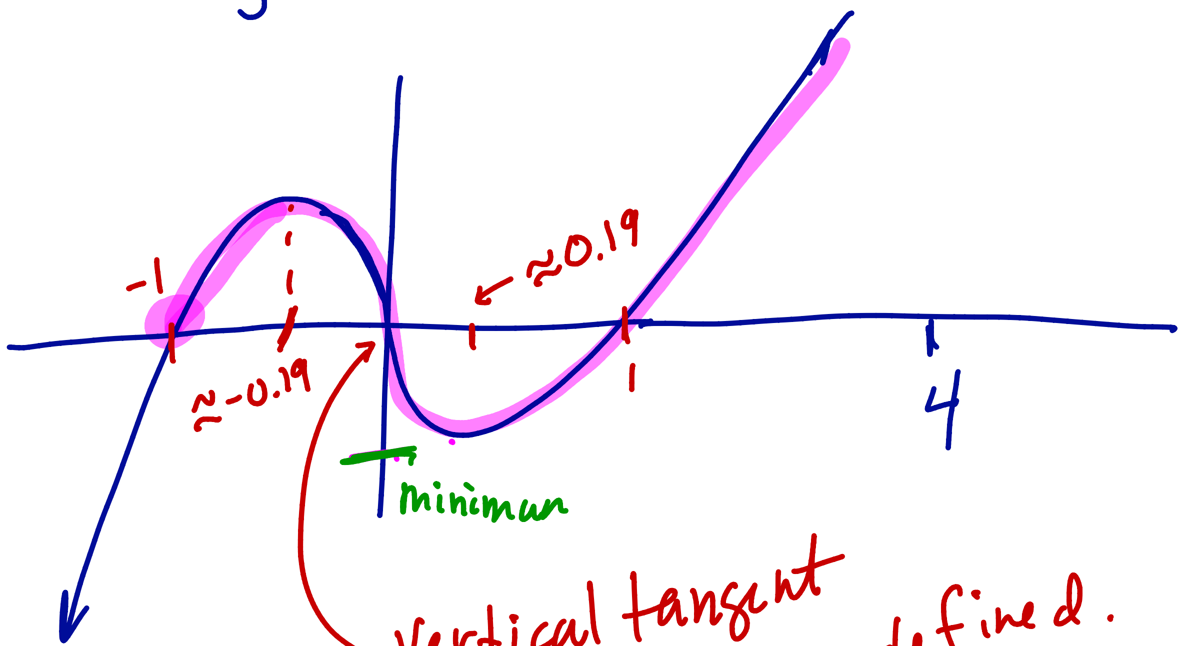
③ ANSWER

max. value $y = 4$ at $x = \pm 8$

min value 0 at $x = 0$

maximum

Rough Sketch of $f(x) = x - \sqrt[3]{x}$ on $[-1, 4]$



Vertical tangent so $f'(0)$ undefined.