SECTION 4.2: THE MEAN VALUE THEOREM

1. Consider the function $f(x) = x^2$ on the interval $[-1, 3]$

   (a) Find the slope of the secant line of the graph of $f(x)$ from $x = -1$ to $x = 3$.

   (b) Find a value of x in $[-1, 3]$ where $f'(x)$ equals the value in part a.

   (c) Make a sketch of the graph of $f(x)$ and add to it the secant line from part a and the tangent line at the location found in part b. What property do the secant line and tangent line have?

2. Repeat Problem 1 with the function $g(x) = 1/x$ on $[1, 5]$.

3. **Mean Value Theorem**

4. What is the *geometric* meaning of the value $\frac{f(b)-f(a)}{b-a}$?

5. Consider the function $f(x) = |x|$ on $[-1, 1]$.

   (a) What would MVT say about f on $[-1, 1]$?

   (b) Does MVT "work" in this case? Why or why not?

6. Suppose f is a continuous function on $[a, b]$ and $f'(x) \geq 0$ for every x in (a, b). How do $f(a)$ and $f(b)$ compare?

7. Suppose f is a continuous function on $[a, b]$ and $f'(x) \leq 0$ for every x in (a, b). How do $f(a)$ and $f(b)$ compare?

8. Compare carefully the following two questions, then answer them.

   (a) Suppose $f(x) = C$ on $[a, b]$, where C is a fixed constant. What can you say about $f'(x)$?

   (b) Suppose $f(x)$ is continuous on $[a, b]$ and $f'(x) = 0$ on (a, b). What can you say about $f(x)$?

9. Suppose a car is traveling down the road and in 30 minutes it travels 32.7 miles. What does the Mean Value Theorem have to say about this?

10. Suppose that $f(0) = -3$ and that $f'(x)$ exists and is less than or equal to 5 for all values of x. How large can $f(2)$ possibly be?

11. **Corollary 7:** If $f'(x) = g'(x)$ for all x in the interval (a, b), then

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Why?