1. Suppose $f(x) = x^5 - 5x^3$.

   (a) Find all critical points of $f(x)$.

   $f'(x) = 5x^4 - 15x^2 = 5x^2(x^2-3) = 0$

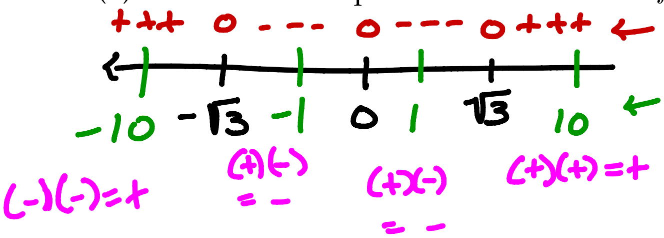
   $\boxed{x = 0, \pm\sqrt{3}}$

   (b) Determine the open intervals on which f is increasing or decreasing.

   will change sign

   ← sign of $f'(x) = 5x^2(x+\sqrt{3})(x-\sqrt{3})$

   always positive

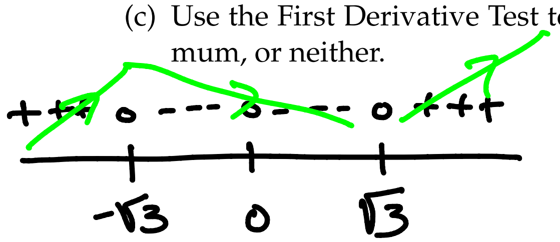


   ← sample points

   ANSWER: f is increasing on $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

   f is decreasing on $(-\sqrt{3}, \sqrt{3})$

   (c) Use the First Derivative Test to classify each critical point as a local minimum, a local maximum, or neither.



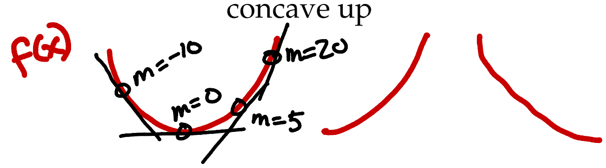
   ANSWER: local max at $x = -\sqrt{3}$

   local min at $x = \sqrt{3}$

   neither at $x = 0$

2. Draw pictures of graphs that are:

   concave up

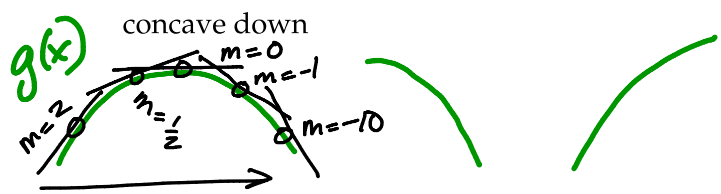


   reading the graph left-to-right, $f'(x)$ is increasing

   If f' is increasing, then

   $\boxed{f'' > 0}$

   concave down



   reading the graph left to right, $f'(x)$ is decreasing

   If f' is decreasing, then

   $\boxed{f'' < 0}$

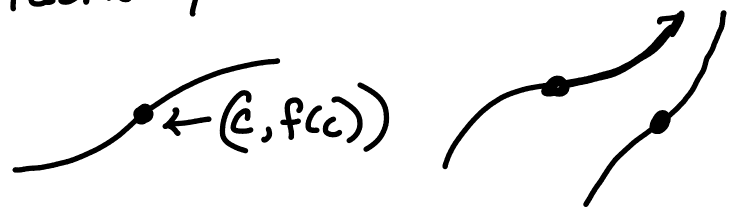
3. What can you conclude about the derivatives of the graphs above?

4. **Concavity Test & Inflection Points**

If $f''(x) > 0$, then $f(x)$ is concave up

If $f''(x) < 0$, then $f(x)$ is concave down.

If $f(x)$ changes concavity at $x = c$ in the domain of $f(x)$, then $(c, f(c))$ is an inflection point.



5. Use the Concavity Test to find the intervals of concavity and the inflection points of the function $f(x) = x^5 - 5x^3$.

$f'(x) = 5x^4 - 15x^2$

$f''(x) = 20x^3 - 30x$

$= \boxed{10x(2x^2-3)} = 0$

$x = 0, \; x = \pm\sqrt{\frac{3}{2}}$

ANSWER:

f is cc up on $(-\sqrt{\frac{3}{2}}, 0) \cup (\sqrt{\frac{3}{2}}, \infty)$

f is cc down on $(-\infty, -\sqrt{3/2})$
$\cup (0, \sqrt{\frac{3}{2}})$

− − − 0 + + + 0 − − − 0 + + + ← sign of f''

−10, $-\sqrt{\frac{3}{2}}$, −1, 0, 1, $\sqrt{3/2}$, 10 ← sample points

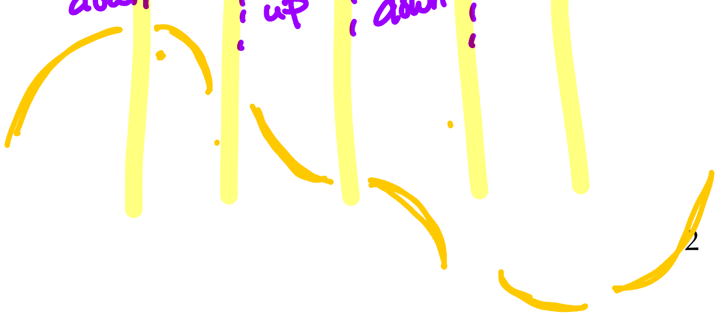
(−)(+) = (+)   (−)(−)=−   (+)(−)=(−)   (+)(+)

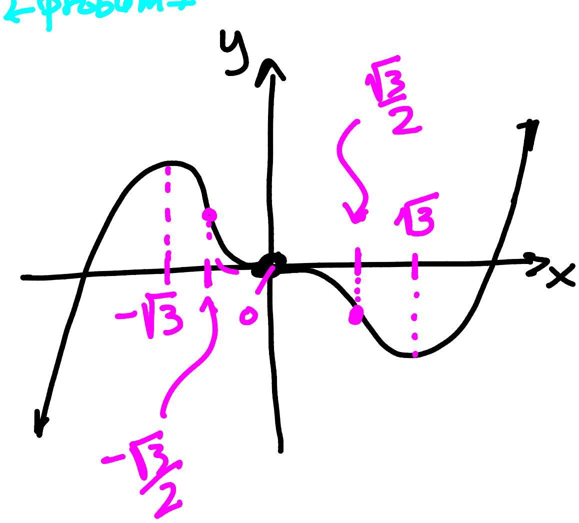
6. Put the information from problems 1 and 5 together to sketch the shape of the graph.

increasing ↑ | decreasing ↓ | decreasing ↓ | increasing ↑ ← problem 1

$-\sqrt{3}$, $-\sqrt{\frac{3}{2}}$, 0, $\sqrt{\frac{3}{2}}$, $\sqrt{3}$

cc down | cc up | cc down | cc up





Suppose $\boxed{x=c \text{ has property that } f'(c)=0}$

7. **The Second Derivative Test of Local Extrema**

cc up

If $\boxed{f''(c)>0}$, then f has a local min at $x=c$

← horizontal tangent

$x=c$

If $\boxed{f''(c)<0}$, then f has a local max at $x=c$

ccdown

$x=c$ horizontal tange

8. Given the function $f(x) = \ln(x^2+4)$ find the following. *For parts a-d, put your answer in a box.*

(a) Determine the domain of $f(x)$. $\mathbb{R}$ or $(-\infty,\infty)$

(b) Find the intervals of increase or decrease.

$$f'(x) = \frac{2x}{x^2+4}.$$

c.p.: $x=0$

-- 0 ++ ← sign of f'

$-1 \quad 0 \quad 1$

answer:

f increasing $(0,\infty)$

f decreasing $(-\infty,0)$

(c) Find the local maximum and minimum values.

--- +++

0

$f(0)=\ln(4)$ is a local minimum.

f has no local max.

(d) Find the intervals of concavity and inflection points.

$$f''(x) = \frac{2(x^2+4)-2x(2x)}{(x^2+4)^2} = \frac{4(2-x^2)}{(x^2+4)^2} = 0 \text{ when } x = \pm\sqrt{2}$$

--- 0 +++ 0 --- ← sign of f''

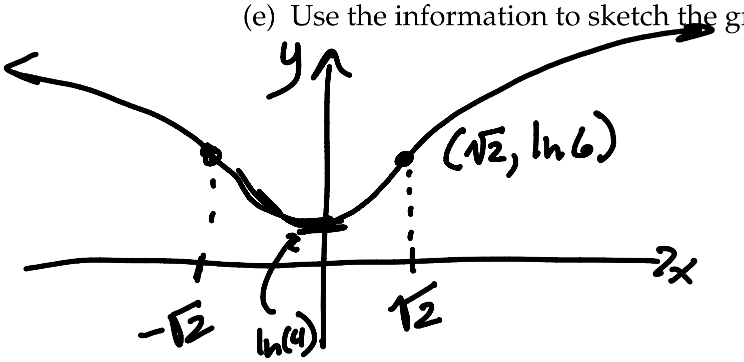
$-100 \quad -\sqrt{2} \quad 0 \quad \sqrt{2} \quad 100$

answer: f is cc up on $(-\sqrt{2},\sqrt{2})$;

f is cc down on $(-\infty,-\sqrt{2}) \cup (\sqrt{2},\infty)$

i. pts: $(\sqrt{2},\ln(6))$, $(-\sqrt{2},\ln 6)$

(e) Use the information to sketch the graph.



Summarize first

9. Sketch a possible graph of a function f that satisfies the following conditions:

   (a) f is continuous and differentiable on $(-\infty, \infty)$. ← one piece, Smooth
   (b) $f(0) = 2$, $f(2) = 3$, $f(4) = 2$ ← points (0,2), (2,3), (4,2)
   (c) $f'(2) = 0$ ← horizontal tangents
   (d) $f'(x) > 0$ for $x < 2$ and $f'(x) < 0$ for $2 < x$
   (e) $f''(x) > 0$ for $4 < x$ and $f''(x) < 0$ for $x < 4$.