4-3 DERIVATIVES AND THE SHAPE OF THE GRAPH (DAY 2)

1. Suppose $f(x) = x^5 - 5x^3$.

   (a) Find all critical points of $f(x)$.

   (b) Determine the open intervals on which f is increasing or decreasing.

   (c) Use the First Derivative Test to classify each critical point as a local minimum, a local maximum, or neither.

2. Draw pictures of graphs that are:

   concave up — concave down

3. What can you conclude about the derivatives of the graphs above?

4. **Concavity Test & Inflection Points**

5. Use the Concavity Test to find the intervals of concavity and the inflection points of the function $f(x) = x^5 - 5x^3$.

6. Put the information from problems 1 and 5 together to sketch the shape of the graph.

7. **The Second Derivative Test of Local Extrema**

8. Use the Second Derivative Test to classify the only critical point of $f(x) = xe^x$. Note $f'(x) = (x+1)e^x$ and $f''(x) = (x+2)e^x$.

9. Sketch a possible graph of a function f that satisfies the following conditions:

   (a) f is continuous and differentiable on $(-\infty, \infty)$.

   (b) $f(0) = 2$, $f(2) = 3$, $f(4) = 2$

   (c) $f'(2) = 0$

   (d) $f'(x) > 0$ for $x < 2$ and $f'(x) < 0$ for $2 < x$

   (e) $f''(x) > 0$ for $4 < x$ and $f''(x) < 0$ for $x < 4$.

10. Given the function $f(x) = \ln(x^2 + 4)$ find the following.

    (a) Determine the domain of $f(x)$.

    (b) Find the intervals of increase or decrease.

    (c) Find the local maximum and minimum values.

    (d) Find the intervals of concavity and inflection points.

    (e) Use the information to sketch the graph.