

SECTION 5-5: SUBSTITUTION (DAY 2)

1. Compute $\int \frac{\sec^2(x)}{\tan(x)} dx = \int \frac{\sec^2 x dx}{\tan x} = \int \frac{du}{u} = \ln|u| + C = \ln|\tan x| + C$

let $u = \tan x$
 $du = \sec^2 x dx$

2. Compute $\int \sec^2(x) \tan(x) dx = \int (\tan x)(\sec^2 x dx) = \int u du = \frac{1}{2} u^2 + C = \frac{1}{2} (\tan x)^2 + C$

let $u = \tan x$
 $du = \sec^2 x dx$

3. Compute $\int \frac{\sin(\theta)}{1+\cos(\theta)} d\theta$

$= \int \frac{\sin \theta d\theta}{1+\cos \theta} = \int \frac{-du}{u} = -\int \frac{du}{u} = -\ln|1+\cos \theta| + C$

let $u = 1 + \cos \theta$
 $du = -\sin \theta d\theta$
 $-du = \sin \theta d\theta$

4. Compute $\int \frac{1}{x \ln(x)} dx = \int \left(\frac{1}{\ln x} \right) \cdot \left(\frac{dx}{x} \right) = \int \frac{1}{u} du = \ln|u| + C$
 $= \ln|\ln x| + C$

let $u = \ln x$
 $du = \frac{1}{x} dx$

5. Compute $\int \frac{\sin(4/x)}{x^2} dx = \int \sin(4x^{-1}) \left(\frac{dx}{x^2} \right) = \int \sin u \left(-\frac{1}{4} du \right)$
 $= -\frac{1}{4} \int \sin u du$
 $= \frac{1}{4} \cos u + C$
 $= \frac{1}{4} \cos(4x^{-1}) + C$

let $u = 4x^{-1}$
 $du = -4x^{-2} dx$
 $-\frac{1}{4} du = \frac{dx}{x^2}$

6. Compute $\int \frac{e^x}{e^x - 3} dx = \int \left(\frac{1}{e^x - 3} \right) (e^x dx) = \int \frac{1}{u} du$
 $= \ln|u| + C$
 $= \ln|e^x - 3| + C$

let $u = e^x - 3$
 $du = e^x dx$

$$7. \text{ Compute } \int \frac{1}{9+x^2} dx = \frac{1}{9} \int \frac{1}{1+\left(\frac{x}{3}\right)^2} dx = \frac{1}{9} \int \frac{1}{1+u^2} (3du)$$

$$\text{let } u = \frac{x}{3}$$

$$du = \frac{1}{3} dx$$

$$3du = dx$$

$$= \frac{1}{3} \int \frac{du}{1+u^2} = \frac{1}{3} \arctan u + C$$

$$= \frac{1}{3} \arctan\left(\frac{x}{3}\right) + C$$

$$8. \text{ Compute } \int \sqrt{x}(x^4+x) dx = \int (x^{9/2} + x^{3/2}) dx = \frac{2}{11} x^{11/2} + \frac{2}{5} x^{5/2} + C$$

$$9. \text{ Compute } \int \cos(x) \sin(\sin(x)) dx = \int (\sin(\sin x)) (\cos x dx)$$

$$\text{Let } u = \sin x$$

$$du = \cos x dx$$

$$= \int \sin(u) du$$

$$= -\cos u + C$$

$$= -\cos(\sin x) + C$$

10. Compute $\frac{d}{dx} [x \ln(x) - x]$. Then compute $\int s^2 \ln(s^3) ds = \int (\ln(s^3))(s^2 ds) =$

$$\frac{d}{dx} [x \ln x - x] = 1 \cdot \ln x + x \cdot \frac{1}{x} - 1 = \ln x + 1 - 1 = \ln x.$$

Let $u = s^3$
 $du = 3s^2 ds$
 $\frac{1}{3} du = s^2 ds$

$$\begin{aligned} &= \frac{1}{3} \int \ln u \, du = \frac{1}{3} (u \ln u - u) + C \\ &= \frac{1}{3} (s^3 \ln(s^3) - s^3) + C \end{aligned}$$

11. Compute $\int x\sqrt{x-1} \, dx = \int (u+1) u^{\frac{1}{2}} du = \int (u^{\frac{3}{2}} + u^{\frac{1}{2}}) du$

Let $u = x-1$
 $u+1 = x$
 $du = dx$

$$\begin{aligned} &= \frac{2}{5} u^{5/2} + \frac{2}{3} u^{3/2} + C \\ &= \frac{2}{5} (x-1)^{5/2} + \frac{2}{3} (x-1)^{3/2} + C \end{aligned}$$

12. Compute $\int_1^3 \frac{(\ln(x))^3}{x} dx = \int_0^{\ln 3} u^3 du = \frac{1}{4} u^4 \Big|_0^{\ln 3}$

Let $u = \ln x$
 $du = \frac{1}{x} dx$
 $x=1, u=0$
 $x=3, u=\ln 3$

$$\begin{aligned} &= \frac{1}{4} ((\ln 3)^4 - 0) \\ &= \frac{(\ln 3)^4}{4} \end{aligned}$$