1. $f(x) = (x-4)\sqrt[3]{x} = x^{4/3} - 4x^{1/3}$; $\quad f'(x) = \frac{4(x-1)}{3x^{2/3}}$; $\quad f''(x) = \frac{4(x+2)}{9x^{5/3}}$.

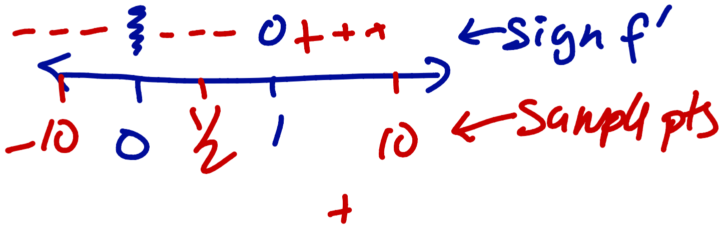
   (a) Find the critical numbers of $f(x)$.

$f' = 0$ when $x = 1$

f' undf. when $x = 0$

$\boxed{\text{answer: } x=0,\ x=1}$

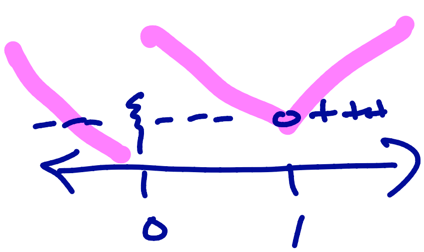
   (b) Find the open intervals on which the function is increasing or decreasing.



f increasing on $(1, \infty)$

decreasing on $(-\infty, 0) \cup (0, 1)$

   (c) Classify all critical points – using the **first derivative test**.



local min at $x = 1$

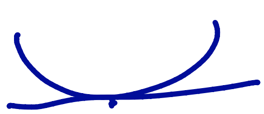
no local max

$x = 0$ is neither a l. min nor l. max

   (d) Classify all critical points – using the **second derivative test**.

$f''(0)$ undefined. (Test fails ☹)

$f''(1) > 0$ so l. min at $x = 1$.



From the other side: $f(x) = (x-4)\sqrt[3]{x} = x^{4/3} - 4x^{1/3}$; $\quad f'(x) = \frac{4(x-1)}{3x^{2/3}}$; $\quad f''(x) = \frac{4(x+2)}{9x^{5/3}}$.

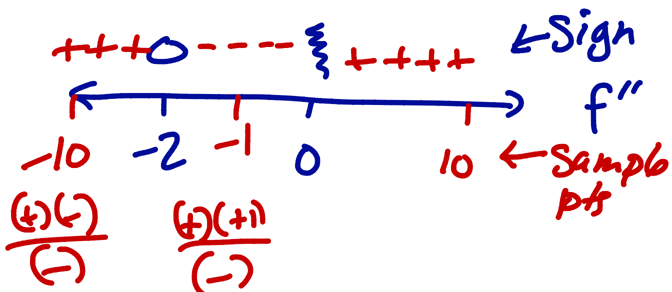
(e) Find the open intervals on which the function is concave up or concave down.

$f''=0$ when $x=-2$

f'' undf. when $x=0$

f is cup on $(-\infty,-2)\cup(0,\infty)$

and ccdown on $(-2,0)$



(f) Find the inflection points.

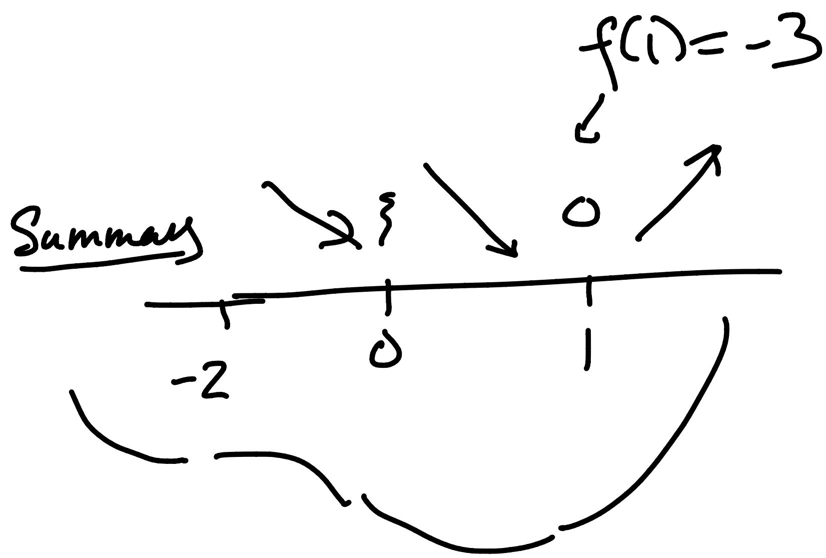
Inflection points at $(-2, \underline{6\sqrt[3]{2}})$ and $(0, \underline{0})$

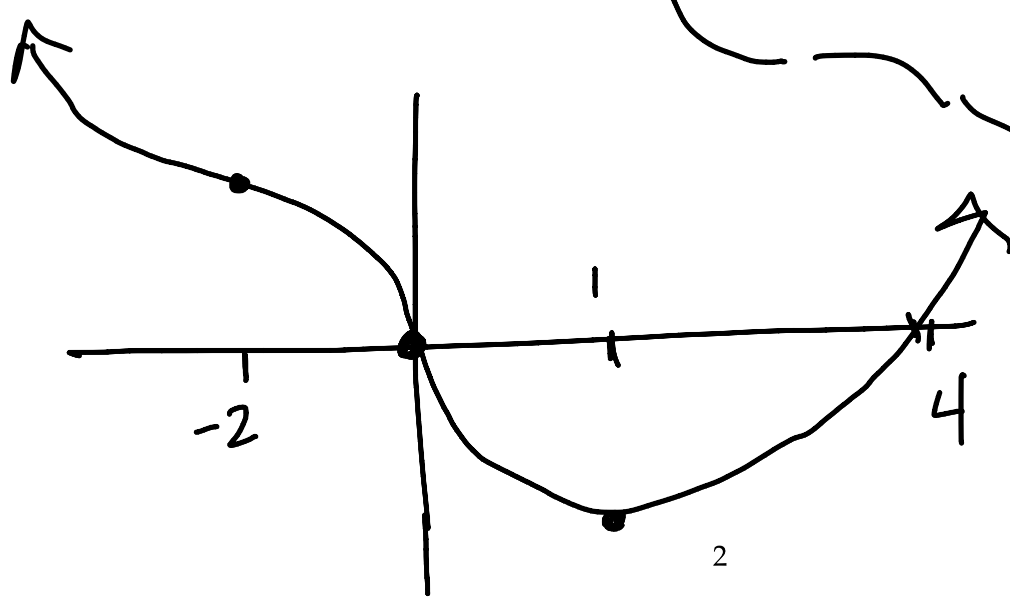
$f(-2) = -6(-2)^{1/3}$

$= 6\sqrt[3]{2}$

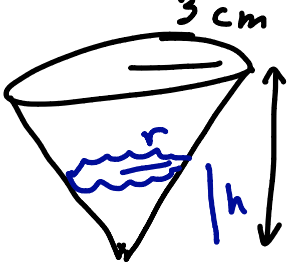
$f(0) = 0$

(g) Sketch the graph.





2. A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of $2\,\text{cm}^3/\text{sec}$, how fast is the water level rising when the water is 5 cm deep? (Volume of a cone is: $V = (1/3)\pi r^2 h$.)



$\frac{dV}{dt} = 2\,\text{cm}^3/\text{s}$

We want $\frac{dh}{dt}$ when $h=5$.

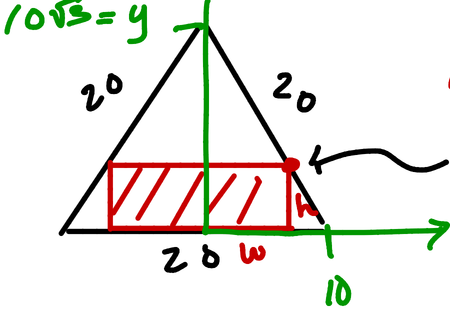
Similar triangles gives: $\frac{r}{h} = \frac{3}{10}$ or $r = \frac{3}{10}h$.

So $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{3}{10}h\right)^2 h = \frac{3\pi}{100}h^3$.

So $\frac{dV}{dt} = \frac{9\pi}{100}h^2\frac{dh}{dt}$. So $\frac{dV}{dt}\cdot\frac{100}{9\pi h^2} = \frac{dh}{dt}$.

Plug in: $\frac{2\cdot 100}{9\pi 5^2} = \frac{dh}{dt}$. Ans: $\frac{dh}{dt} = \frac{8}{9\pi}$ cm/s

3. Find the dimensions of the rectangle of maximum area that can be inscribed in an equilateral triangle of side 20 cm if one side of the rectangle likes on the base of the triangle.



maximize area $A = 2wh$

line: $m = \frac{-10\sqrt{3}}{10} = -\sqrt{3}$, point $(10,0)$

$y - 0 = -\sqrt{3}(x-10)$

$y = -\sqrt{3}x + 10\sqrt{3}$

$y^2 + 10^2 = 20^2$

$y = \sqrt{300} = 10\sqrt{3}$

But (w,h) lies on line So $h = -\sqrt{3}w + 10\sqrt{3}$.

So $A(w) = 2w\left(-\sqrt{3}w + 10\sqrt{3}\right) = -2\sqrt{3}w^2 + 20\sqrt{3}w$ on $[0,10]$

$A'(w) = -4\sqrt{3}w + 20\sqrt{3} = 0$

So $w = 5$. ANS: width: 10, height: $5\sqrt{3}$

4. Determine the absolute maximum and absolute minimum of $f(x) = \frac{1}{x} + x$ on $[1/2, 4]$.

$f(x) = x^{-1} + x$

$f'(x) = -x^{-2} + 1 = 0$

$1 = \frac{1}{x^2}$

So $x = \pm 1$. (only +1 in domain)

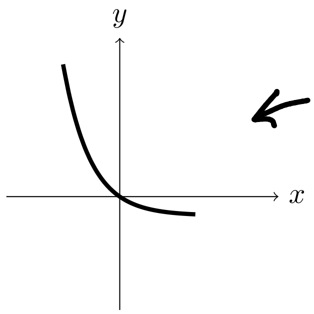
abs max: 4.25 at $x=4$

abs min: 2 at $x=1$

table

x	$\frac{1}{2}$	1	4
$f(x)$	$2+\frac{1}{2} = 2.5$	2	$\frac{1}{4}+4=4.25$

5. Below is the graph of the DERIVATIVE, $h'(x)$ of $h(x)$.



← $h'(x)$

(a) Does $h(x)$ have any critical points? If so, what are they?

c.p. $x=0$

(b) Does $h(x)$ have any local extrema? If so, where do they occur and what type (min/max)?

$+ \quad - \quad$ ← sign $h'(x)$

0

local max at $x=0$

(c) Can you determine if $h(x)$ is concave up or down?

h' is decreasing. So $h'' < 0$. So h is ccdown.

6. A circular metal plate is being heated in an oven. The radius of the plate is increasing at a rate of 0.01 cm/min when the radius is 50cm. How fast is the area of the plate increasing?

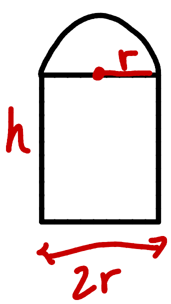
Find $\frac{dA}{dt}$ when $r=50$.

$A=\pi r^2$

$\frac{dA}{dt}=2\pi r\frac{dr}{dt}$

$\frac{dA}{dt}=2\pi(50)(0.01)=\pi\ cm^2/min$

7. A Norman window is has a rectangular base and a semi-circle on top. What dimensions of the window minimize the perimeter if the area of the window is to be 4 ft^2.



minimize perimeter.

Area $= A = 4ft^2$

$4=\frac{1}{2}\pi r^2+2rh \xrightarrow{\text{Solve for } h} h=\left(4-\frac{1}{2}\pi r^2\right)/2r=\frac{8-\pi r^2}{4r}$

$P=\frac{1}{2}(2\pi r)+2r+2h$

$P(r)=2r+\pi r+\frac{8-\pi r^2}{2r}$

$=\frac{4}{r}+\frac{\pi}{2}r+2r \quad [0,\infty)$

$P'(r)=-4r^{-2}+\frac{\pi}{2}+2=0$

$r=\frac{4}{2+\pi/2}=\frac{8}{4+\pi}$

$+ \quad 0 \quad -$

$0 \qquad \frac{8}{4+\pi}$

max @ $r=\frac{8}{4+\pi}$

8. Find the domain of the function $f(x)=\frac{\sin(5x)}{x^2+x}$ and identify any vertical or horizontal asymptotes. Justify your answers.

$f(x)=\frac{\sin(5x)}{x(x+1)}$ domain: $(-\infty,-1)\cup(-1,0)\cup(0,\infty)$

H.A. : $\lim_{x\to\infty}\frac{\sin(5x)}{x(x+1)}=0$. So $y=0$

V.A. $\lim_{x\to 0}\frac{\sin(5x)}{x^2+x}\overset{H}{=}\lim_{x\to 0}\frac{5\cos(5x)}{2x+1}=\frac{5}{1}\neq\pm\infty$. So $x=0$ NOT asymptote.

$\lim_{x\to -1}\frac{\sin(5x)}{x^2+x}=\pm\infty$ since $\sin(5x)\to\sin(-5)$ and $x^2+x\to 0^{+/-}$

So $x=-1$ is a vertical asymptote.