SECTION 4.8 NEWTON'S METHOD

1. Newton's Method is an iterative rule for finding roots.

   Given: $F(x)$
   Want: a so that $F(a) = 0$
   Guess: x_0 close to a

   Plug in and Repeat:

2. Let $F(x) = x^2 - 2$.

   (a) Using elementary algebra, find a such that $F(a) = 0$. (Find a exactly and find a decimal approximation with at least 9 decimal places.)

   (b) Find a formula for x_{k+1}. Simplify it.

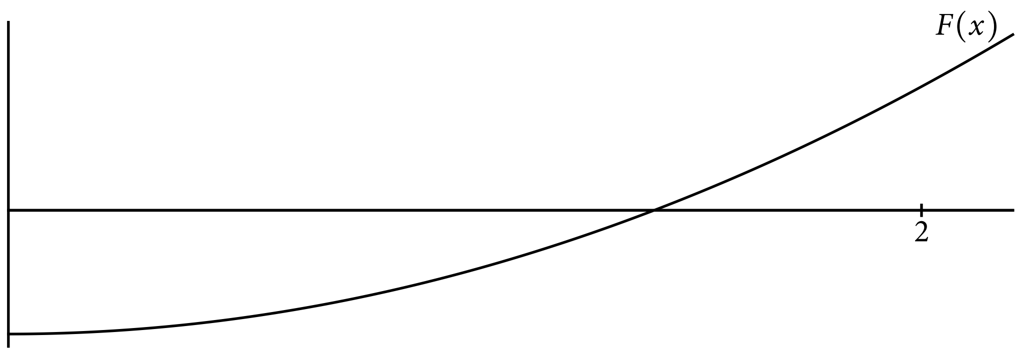
   (c) Using an initial guess of $x_0 = 2$, complete 4 iterations of Newton's method to find x_4 and compare your answer to the one in part (a).

3. This page is intended to illustrate *how* Newton's Method works and *why* it has the formula it does.

Again, consider the function $F(x) = x^2 - 2$.

(a) Find the linearization $L(x)$ of $F(x)$ at $x = 2$. Leave your answer in point-slope form.

(b) I've graphed $F(x)$ for you below. Mark where $\sqrt{2}$ is on this diagram and add to this diagram the graph of $L(x)$.



(c) Find the number x_1 such that $L(x_1) = 0$.

(d) In the diagram above, label the point x_1 on the x-axis.

(e) Let's do it again! Find the linearization $L(x)$ of $F(x)$ at $x = x_1$.

(f) Add the graph of this new linearization to your diagram on the first page.

(g) Find the number x_2 such that $L(x_2) = 0$. Then label the point $x = x_2$ in the diagram.

(h) Compare your numbers for x_1 and x_2 to those on the previous page. They should be the same.

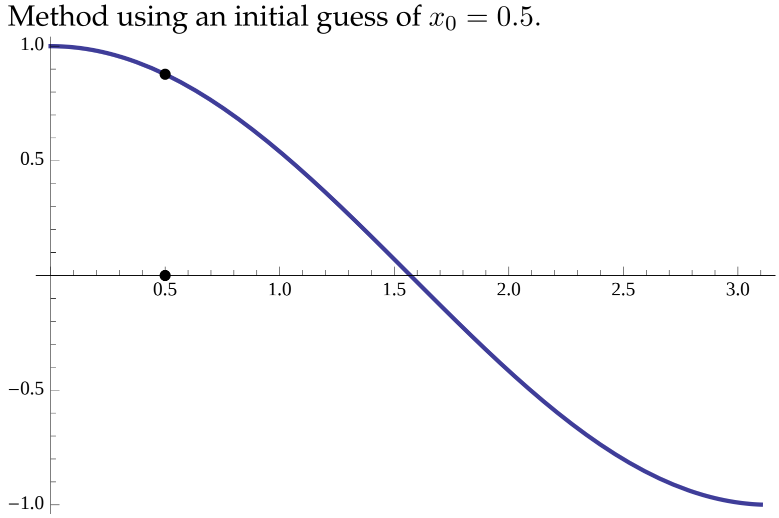
(i) Let's be a little more systematic. Suppose we have an estimate x_k for $\sqrt{2}$.

- Compute $F(x_k)$.
- Compute $F'(x_k)$.
- Compute the linearization of $F(x)$ at $x = x_k$.

$L(x) =$

- Find the number x_{k+1} such that $L(x_{k+1}) = 0$. You should try to find as simple an expression as you can. Compare this to the formula we used on problem 1b from page 1.

4. Indicate on the picture below, the values of x_1, x_2 and x_3 that would be obtained from Newton's Method using an initial guess of $x_0 = 0.5$.



5. Try to solve

$$e^{-x} - x = 0$$

by hand.

6. Explain why there is a solution between $x = 0$ and $x = 1$.

7. Starting with $x_0 = 1$, find an approximation of the solution of $e^{-x} - x = 0$ to 6 decimal places. During your computation, keep track of each x_k to at least 9 decimal places of accuracy.