

## SECTION 4.9 ANTIDERIVATIVES

1. Find a particular antiderivative of  $f(x) = 9 + x - x^2$ .

$$F(x) = 9x + \frac{1}{2}x^2 - \frac{1}{3}x^3$$

2. Find all antiderivatives of  $f(x) = 9 + x - x^2$ .

$$F(x) = 9x + \frac{1}{2}x^2 - \frac{1}{3}x^3 + C, \quad C - \text{general constant}$$

3. Find an antiderivative of  $f(x) = \frac{1}{x^2} = x^{-2}$

$$F(x) = -x^{-1}$$

← Not just +C

4. To find *all* antiderivatives of a function  $f(x)$ , do you always just add a +C?

No

Example:

$$F(x) = \begin{cases} -\frac{1}{x} + 10 & x > 0 \\ -\frac{1}{x} - \pi & x < 0 \end{cases}$$

5. For each of the following functions, find a particular antiderivative.

Function	Antiderivative	Function	Antiderivative
$x$	$\frac{1}{2}x^2$	$\sin(x)$	$-\cos x$
$x^2$	$\frac{1}{3}x^3$	$\cos(x)$	$\sin x$
$x^3$	$\frac{1}{4}x^4$	$e^x$	$e^x$
$x^k \ (k \neq -1)$	$\frac{1}{k+1} x^{k+1}$	$1/(1+x^2)$	$\arctan x$
$x^{-1}$ for $x > 0$	$\ln(x)$	$\sec^2(x)$	$\tan x$
$x^{-1}$ for $x < 0$	$\ln(-x)$	$\sec(x) \tan(x)$	$\sec x$
$x^{-1}$ for all $x$	$\ln( x )$	$1$	$x$

6. Compute <sup>an</sup> ~~three different~~ antiderivatives of  $f(x) = 15x^{20} + 44x^{10} + 8$

$$F(x) = \frac{15}{21} x^{21} + 4x^{11} + 8x$$

7. Compute an antiderivative of  $f(t) = \frac{5 \sec t \tan t}{3} - 4 \sin t - \frac{1}{t} + e^2$

$$F(x) = \frac{5}{3} \sec t + 4 \cos t - \ln|x| + e^2 x$$

8. Compute an antiderivative of  $f(x) = \cos(3x)$ .

$$F(x) = \frac{1}{3} \sin(3x)$$

9. Compute the antiderivative of  $f(t) = t^2$  that equals 5 when  $t = 2$ .

$$F(t) = \frac{1}{3} t^3 + C$$

$$F(2) = 5 = \frac{1}{3} (2)^3 + C$$

$$\text{So } C = 5 - \frac{8}{3} = \frac{7}{3}$$

$$\text{Answer: } F(t) = \frac{1}{3} t^3 + \frac{7}{3}$$