Lecture Notes: Review for Final Exam (day 1)

The Final Exam will be cumulative. The exam will be designed to be completable in 2 hours. Books, notes, calculators and other aids are not allowed. As with all assessments in this course, you are strongly encouraged to work some old Final Exams as practice.

Sample Problems

1. Given $f(x) = 3x - x^2$, find $f'(x)$ *using the definition of the derivative.*

2. Find the equation of the line tangent to $ye^x + 2 = x^2 + y^2$ at the point $(0, 2)$.

3. Let $F(t) = \frac{20}{4+e^{-2t}}$ model the population of fish in hundreds of fish, where time t is measured in years.

   (a) Find and interpret $F(0)$.

   (b) Find and interpret (in language your non-quantitative friends could understand) $\lim_{t\to\infty} F(t)$.

   (c) Find $F'(t)$. (HINT: You can check your answer with the one at the bottom of the page.)

   (d) Find and interpret $F'(0)$.

   (e) Find and interpret (in language your non-quantitative friends could understand) $\lim_{t\to\infty} F'(t)$.

   (f) Give a rough sketch the graph of $F(t)$ given the information above.

HINT: $F'(t) = \frac{40e^{-2t}}{(4+e^{-2t})^2}$

4. Let $f(x) = \frac{5x^2}{1-\cos(x)}$.

   (a) Find $\lim_{x \to 0} \dfrac{5x^2}{1 - \cos x}$

   (b) Does $f(x)$ have a vertical asymptote at $x = 0$? Explain

5. Let $g(x) = \frac{4x^4+5}{(x^2-2)(2x^2-1)}$. Does $g(x)$ have any horizontal asymptotes? Justify your answer with a limit.

6. Complete two iterations of Newton's Method to estimate a solution to $x^7 + 4 = 0$. Use $x_0 = -1$. (Note you may leave your second iteration in unsimplified form.)

7. Evaluate.

(a) $\displaystyle\int_0^{\pi/4} \frac{\sec^2 t}{\tan(t)+1}\,dt$

(b) $\displaystyle\int_0^{8} \frac{3}{\sqrt{x+1}}\,dx$

8. A particle is moving with velocity $v(t) = 2t - \frac{1}{1+t^2}$ measured in meters per second.

(a) Find and interpret $v(0)$.

(b) Find the displacement for the particle from time $t = 0$ to time $t = 4$. Give units with your answer.

(c) If D is the *distance* the particle traveled over the interval $[0, 4]$, is D larger or smaller or exactly the same as your answer in part (b)? Justify your answer.

(d) Assuming $s(0) = 1$, find the position of the particle at any time t.