Logistics:

- It covers material from Chapters 1 and 2, though Chapter 2 is emphsized.
- One-hour in length.
- No books, notes, calculators or other aids.
- Students in the first class will stay in class till the hour is over. Bring a book or other homework. No phones for the entire hour.
- Multiple versions of the midterm. All will be graded the same way by the same people.
- There is an extra-credit question.
- You cannot re-do a midterm.
- Using your cell phone or computer for any reason during the Midterm will result in a grade of zero.

Things to Keep in Mind:

- *All* of the problems will be familiar to any student who attended class, did the homework, and took the quizzes.
- **Be a good test taker.** If you are stuck on a problem, leave it and come back. Pay attention to the time and make sure you get to every problem. Attack each problem under the assumption that you have the tools to solve it. If time permits, check every problem by working it a different way or by checking the plausibility of your answer.
- **Be active, not passive,** when reviewing for this (and all) midterms. It is better to work problems and/or take a practice test, then to read the book, read over your notes, or look over solutions to quizzes/tests/homework.
- Don't skip Recitation. It will be an active review. Focus on your weak areas.
- Is there any good thing about preparing for and taking a Midterm?

Topic Review Chapter 2

1. Identify all of the following given a picture of the graph of $f(x)$.
   - domain, range, regions where the function is (or isn't) continuous, points where the derivative of the function fails to exist.
   - limits of various kinds (infinite, one-sided, two-sided)
   - values of the function (Given x, find y. Given y, find x.)

2. Evaluate limits algebraically.
   Recall the various types: one-sided, two-sided, infinite, at infinity
   Recall various strategies: get a common denominator, factor and cancel, rationalize, divide by the highest power of x in the denominator, Squeeze Theorem, and more
   But *when* to use these tricks?

   Example: Let $f(x) = \frac{x^2-1}{2x^2+3x+1}$.
   Consider the limit of $f(x)$ as $x \to 1$, $x \to -1$, $x \to -1^+$, $x \to -\infty$.
   What about the limit of $(x+1)e^{f(x)}$ as $x \to 0$?

3. Understand the relationship between limits and asymptotes.

4. Know the definition of continuity and *how to use it to show a function is or is not continuous at a point.*

   Example: Use the definition to show that $f(x) = \begin{cases} x & x \leq 10 \\ 2x + 10\cos((x-1)\pi) & 10 < x \end{cases}$ is continuous at $x = 10$.

5. Know how to find the derivative of $f(x)$ at $x = a$ using the definition.
   Example: $f(x) = 2\sqrt{x}$ at $x = 9$

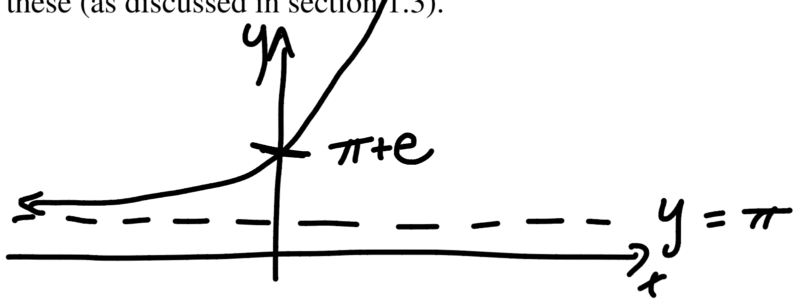
6. Know how to *interpret* the derivative of $f(x)$ at $x = a$.
   - as the slope of the tangent line to the graph of $f(x)$ at $x = a$
   - as instantaneous rate of change of f with respect to x at $x = a$.

7. Know what the Intermediate Value Theorem says.

Chapter 1

1. Know how to graph the catalog of essential functions (as discussed in section 1.2) and transformations of these (as discussed in section 1.3).

   Graph $f(x) = \pi + e^{x+1}$



Example: Let $f(x) = \frac{x^2-1}{2x^2+3x+1}$.
Consider the limit of $f(x)$ as $x \to 1$, $x \to -1$, $x \to -\frac{1}{2}$, $x \to -\infty$.
What about the limit of $(x+1)e^{f(x)}$ as $x \to 0$?

$x \to -\frac{1}{2}^+$

$$\lim_{x \to 1} \frac{x^2-1}{2x^2+3x+1} = \frac{0}{6} = 0$$

$$\lim_{x \to -1} \frac{x^2-1}{2x^2+3x+1} = \lim_{x \to -1} \frac{(x+1)(x-1)}{(2x+1)(x+1)} = \lim_{x \to -1} \frac{x-1}{2x+1} = \frac{-2}{-1} = 2$$

$\frac{0}{0}$

$$\lim_{x \to -\frac{1}{2}} \frac{x^2-1}{2x^2+3x+1} = \lim_{x \to -\frac{1}{2}} \frac{x-1}{2x-1} = \text{DNE}$$

$$\lim_{x \to -\frac{1}{2}^+} \frac{x^2-1}{2x^2+3x+1} = \lim_{x \to -\frac{1}{2}^+} \frac{x-1}{2x-1} = -\infty$$

Vertical asymptote at $x=-\frac{1}{2}$

$x \to -\frac{1}{2}^+$, $x-1 \to -\frac{3}{2}$; $2x-1 \to 0^+$

$$\lim_{x \to -\infty} \frac{x^2-1}{2x^2+3x+1} = \lim_{x \to -\infty} \frac{1-\frac{1}{x^2}}{2+\frac{3}{x}+\frac{1}{x^2}} = \frac{1}{2}$$

horizontal asymptote at $x=\frac{1}{2}$

Example: Use the definition to show that $f(x) = \begin{cases} x & x \leq 10 \\ 2x + 10\cos((x-1)\pi) & 10 < x \end{cases}$

is continuous at $x = 10$.

1. $f(10) = 10$

2. $\lim_{x \to 10^-} f(x) = \lim_{x \to 10^-} x = 10$,

3. $\lim_{x \to 10^+} 2x + 10\cos((x-1)\pi) = 20 - 10 = 10$

4. So $\lim_{x \to 10} f(x) = 10$ 5. $= f(10)$.

Example: $f(x) = 2\sqrt{x}$ at $x = 9$

$$f'(9) = \lim_{h\to 0} \frac{f(9+h) - f(9)}{h} = \lim_{h\to 0} \frac{2\sqrt{9+h} - 6}{h} \cdot \frac{(2\sqrt{9+h} + 6)}{(2\sqrt{9+h} + 6)}$$

$$= \lim_{h\to 0} \frac{4(9+h) - 36}{h(2\sqrt{9+h} + 6)} = \lim_{h\to 0} \frac{4h}{h(2\sqrt{9+h} + 6)}$$

$$= \lim_{h\to 0} \frac{4}{2\sqrt{9+h} + 6} = \frac{4}{12} = \frac{1}{3}$$

$-3+6$

Interpretations:

① slope: $m = \frac{1}{3}$. line: $y - 6 = \frac{1}{3}(x-9)$

$y = \frac{1}{3}x + 3$

② rate of change: y = distance in ft

x = time in sec

$f'(9) = \frac{1}{3}$ means object moving w/ instantaneous velocity of $\frac{1}{3}$ ft/sec at 9 seconds into experiment.

2. Know how to find the domain and range of functions and how to solve equations. Good example problems: Section 1.5 # 51-54, Review Problems (page 70) #5-8, 25-26.

   Find the domain of the function $f(x) = \frac{x^2}{2+3\ln(x)}$.

We need $2+3\ln(x) \neq 0$.

So, solve:

$2+3\ln x = 0$

$\ln x = -\frac{2}{3}$

$x = e^{-2/3}$.

We need $x>0$ so that $\ln(x)$ is defined.

Answer: $D: (0, e^{-2/3}) \cup (e^{-2/3}, \infty)$