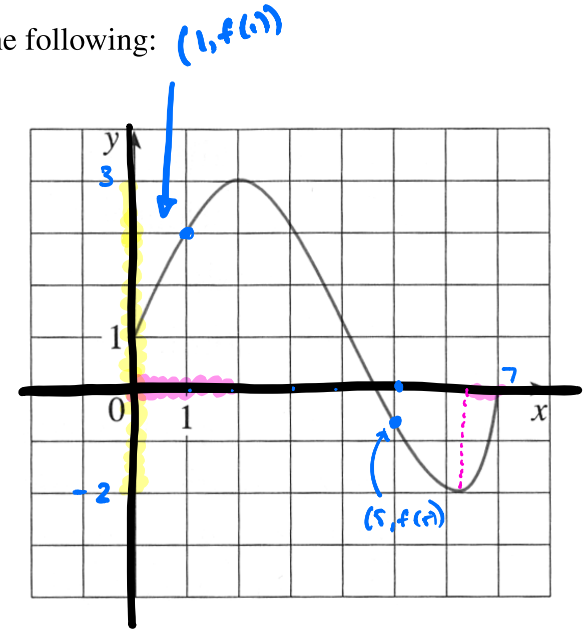
1. The graph of a function f is shown below. Find the following:



a) $f(1)$ and $f(5)$

$f(1) = 3; \quad f(5) \approx -2/3$

b) the domain of f

Where is f defined? On the interval $[0, 7]$

c) the range of f

What y-values does f attain? $[-2, 3]$

d) For which value of x is $f(x) = 4$?

None!

e) Where is f increasing?

$[0, 2] \cup [6\tfrac{1}{3}, 7]$

2. Let $f(x) = 3x^2 - x + 2$. Find and simplify the following expressions. Are (b) and (c) different?

(a) $f(2)$

$f(2) = 3(2)^2 - 2 + 2 = 3 \cdot 4 = 12$

(b) $f(a^2)$

$f(a^2) = 3(a^2)^2 - (a^2) + 2 = 3a^4 - a^2 + 2$

(c) $[f(a)]^2$

$(f(a))^2 = (3a^2 - a + 2)^2 = (3a^2 - a + 2)(3a^2 - a + 2)$

$= (3a^2)(3a^2) + (3a^2)(-a) + (3a^2)(2) + (-a)(3a^2) + (-a)(-a) + (-a)(2)$

$+ 2(3a^2) + 2(-a) + 2(2) = 9a^4 - 3a^3 + 6a^2 - 3a^3 + a^2 - 2a + 6a^2 - 2a + 4$

$= 9a^4 - 6a^3 + 13a^2 - 4a + 4$

We will be working with this type of expression a lot!

(d) $\dfrac{f(a+h) - f(a)}{h}$

$\dfrac{f(a+h) - f(a)}{h} = \dfrac{1}{h}\left(\left[3(a+h)^2 - (a+h) + 2\right] - \left[3a^2 - a + 2\right]\right)$

$= \dfrac{1}{h}\left(3(a^2 + 2ah + h^2) - \cancel{a} - h + \cancel{2} - 3a^2 + \cancel{a} - \cancel{2}\right) = \dfrac{1}{h}\left[3\cancel{a^2} + 6ah + 3h^2 - h - 3\cancel{a^2}\right]$

$= \dfrac{1}{h}(6ah + 3h^2 - h) = 6a + 3h - 1$

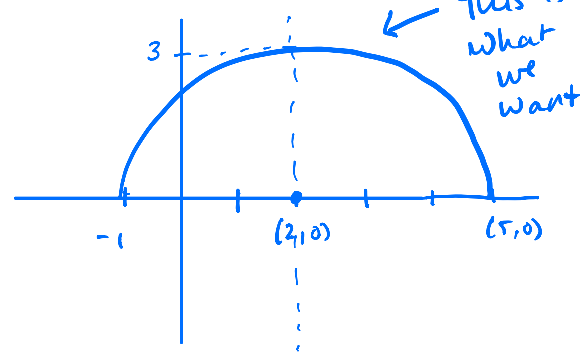
3. Write a formula for the top half of the circle with center $(2, 0)$ and radius 3.

Circle is $(x-2)^2 + y^2 = 3^2$

(check: when we put in 2, what do we get out?)

So upper half-circle is

$$y = \sqrt{3^2 - (x-2)^2}$$



4. Find the domain of each of the following functions. Use interval notation.

(a) $f(x) = \dfrac{1}{x^2 - 16}$

← We're asking the question: which $\underline{x}$ values make the function be undefined?

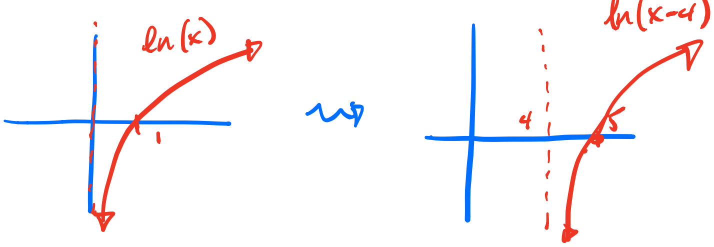
$f(x) = \dfrac{1}{(x-4)(x+4)}$ ← We can't divide by zero! So we must exclude $x = 4$ and $x = -4$.

domain is $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

As a picture:

(b) $g(x) = \ln(x - 4)$

We know that $\ln(x)$ is defined for $x > 0$, so $\ln(x-4)$ is defined for $x - 4 > 0$ $\Rightarrow x > 4$.



So the domain is $(4, \infty)$

5. Graph the piecewise defined function.

$$f(x) = \begin{cases} x + 1 & \text{if } x \le -1 \\ x^2 & \text{if } x > -1 \end{cases}$$

When $x \leq 1$, $f(x) = x + 1$, which is a line, passing through $(-1, 0)$ with a slope of 1

When $x > -1$, $f(x) = x^2$

at $x = -1$, this would be $(-1, 1)$ but it doesn't quite get there! Hence the open circle.