Transformation Review

1. Explain what each does to the *original* graph $y = f(x)$. (Assume $c > 0$.)

   (a) $f(x) + c$ Shifts ↑ by c

   (b) $f(x) - c$ Shifts ↓ by c

   (c) $f(x + c)$ Shifts ← by c

   (d) $f(x - c)$ Shifts → by c

   (e) $cf(x)$ Scales by c ↕

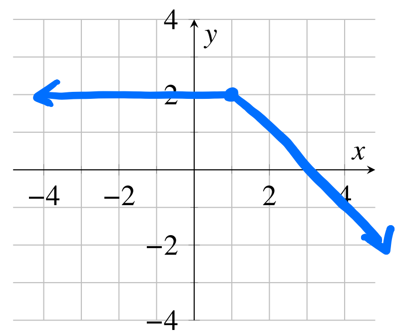
   (f) $f(cx)$ compresses by c ↔

   (g) $-f(x)$ flips ↕

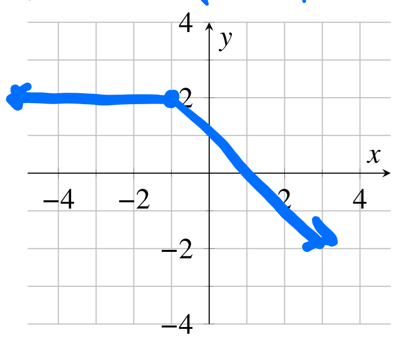
   (h) $f(-x)$ flips ↔

2. Let $f(x) = \begin{cases} 2 & x \le 1 \\ 3 - x & x > 1 \end{cases}$. Graph each of the following *using the ideas from # 1 above.*

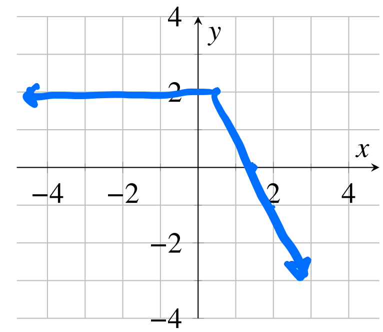
   (a) $f(x)$



   (b) $f(x + 1)$ Shift ← by 2

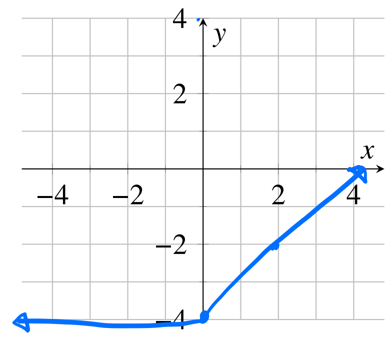


   (c) $f(2x)$ Compress by 2 ↔



Check: at x=2
f(2x) =
3-4 = -1 ✓

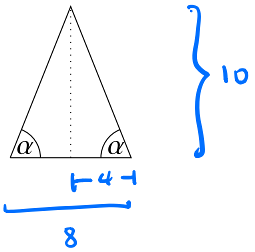
   (d) $-2f(x)$ flip ↕ and expand by 2 ↕

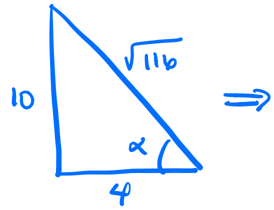


Check: at x = 2, -2f(x) =
-2(3-2) = -2

Trigonometry Review

3. An isosceles triangle has a height of 10 ft and its base is 8 feet long. Determine the sine, cosine, tangent, cotangent, secant and cosecant of the base angle α.





$\Longrightarrow$

$\sin(\alpha) = \frac{10}{\sqrt{116}} = \frac{5}{\sqrt{29}}$

$\cos(\alpha) = \frac{4}{\sqrt{116}} = \frac{2}{\sqrt{29}}$

$\tan(\alpha) = \frac{10}{4} = \frac{5}{2}$

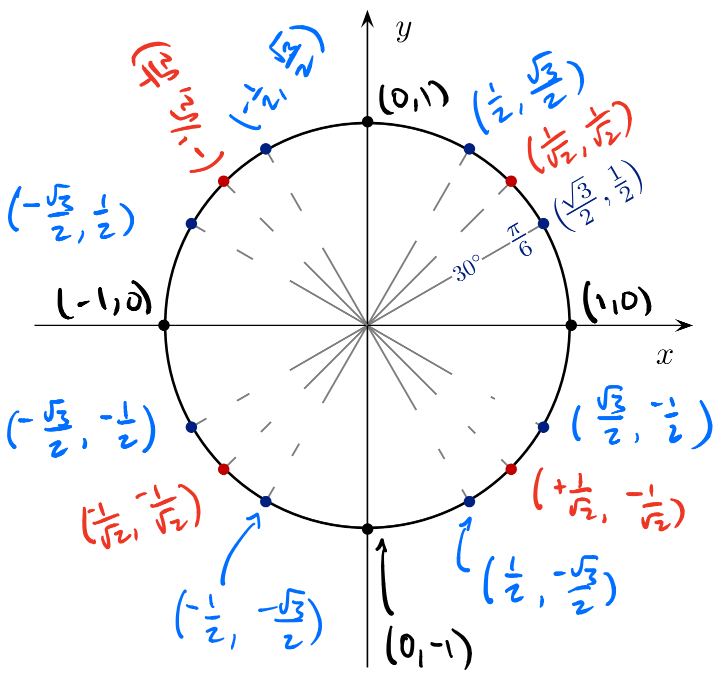
$\csc(\alpha) = \frac{\sqrt{116}}{10} = \frac{\sqrt{29}}{5}$

$\sec(\alpha) = \frac{\sqrt{116}}{4} = \frac{\sqrt{29}}{2}$

$\cot(\alpha) = \frac{4}{10} = \frac{2}{5}$

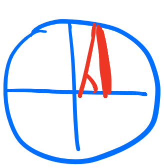
and hypotenuse is $\sqrt{4^2+10^2} = \sqrt{116} = 2\sqrt{29}$

4. Using a 45-45-90 triangle and a 30-60-90 triangle find the coordinates of **any three marked points, one of each color** on the unit circle. (The blue points are at multiples of $\frac{\pi}{6}$, the red points are at multiples of $\frac{\pi}{4}$, and the black points are at multiples of $\frac{\pi}{2}$.)



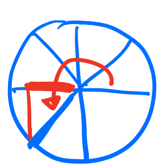
5. Without a calculator evaluate:

(a) $\sin(\frac{2\pi}{3})$

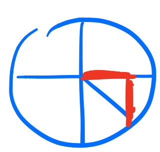


$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

(b) $\cos(\frac{5\pi}{4})$ $= -\frac{1}{\sqrt{2}}$

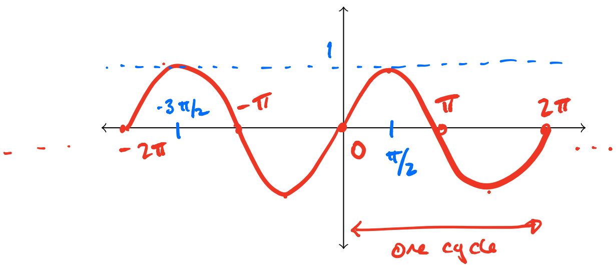


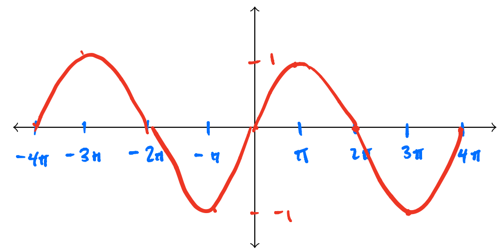
(c) $\tan(\frac{-\pi}{4})$ $= \frac{-1/\sqrt{2}}{1/\sqrt{2}} = -1$



6. On the axes below, graph *at least two cycles* of $f(x) = \sin x$, $f(x) = \sin(x/2)$. Label all x- and y-intercepts.

$f(x) = \sin(x)$





here, we are expanding by 2 (= compressing by 1/2)
Check: at $x = \pi$, $\sin(\frac{1}{2} \cdot \pi) = \sin(\frac{\pi}{2}) = 1$

7. (a) Use the graph of $f(x) = \sin(x)$ to solve $\sin(x) = 1$

Need where $\sin(x) = 1 \implies$

$x = \frac{\pi}{2} + k(2\pi)$ for $\underbrace{\text{integers } k}_{k \in \mathbb{Z}}$

(b) Use the graph of $f(x) = \sin(x/2)$ to determine the domain of $f(x) = \csc(x/2)$

From the graph, we see that $\sin(x/2) = 0$ at integer multiples of 2π. Therefore, the domain is

$$\{ \underbrace{x \in \mathbb{R}}_{\text{real \#s } x} : \underbrace{x \neq 2\pi k \text{ for } k \in \mathbb{Z}}_{} \}$$

set of all

real #s x, where the set of real numbers is denoted $\mathbb{R}$

such that

x satisfies this property, that x is not equal to $2\pi k$

k in the set of integers, which is represented as $\mathbb{Z}$