Evaluate each limit. Show your work or explain your reasoning.

1. $\lim_{h\to 0} \frac{(-9+h)^2-81}{h}$

← We cannot use direct substitution because the denominator $\to 0$ as $h \to 0$

$= \lim_{h\to 0} \frac{1}{h}(81 - 18h + h^2 - 81)$

$= \lim_{h\to 0} \frac{1}{h}(-18h + h^2)$

$= \lim_{h\to 0} -18 + h = -18$

2. $\lim_{t\to 8}(1+\sqrt[3]{t})(2-t^2)$ We can use direct substitution here.

$= \lim_{t\to 8}(1+\sqrt[3]{t}) \cdot \lim_{t\to 8}(2-t^2)$

$= (1+\sqrt[3]{8})(2-8^2)$

$= (1+2)(2-64)$

$= 3(62)$

$= 186$

$$\begin{array}{r} 62 \\ 3 \\ \hline 186 \end{array}$$

3. $\lim_{\theta\to 4} \frac{\theta^2-4\theta}{\theta^2-\theta-12}$

Note $4^2 - 4 - 12 = 16 - 16 = 0$. "type" 0/0. Need algebra tricks!

$= \lim_{\theta\to 4} \frac{\theta(\theta-4)}{(\theta-4)(\theta+3)}$

$= \lim_{\theta\to 4} \frac{\theta}{\theta+3}$

$= \frac{4}{4+3}$

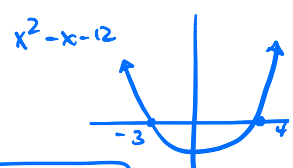
$= 4/7$

4. $\lim_{x\to 4} \frac{x^2}{x^2-x-12}$ ($x^2 \to 16$)

As $x \to 4$, $x^2 - x - 12 \to 0$ and $x^2 \to 16$

$\lim_{x\to 4^-} \frac{x^2 \to 16}{x^2-x-12 \to 0^-} = -\infty$

$\lim_{x\to 4^+} \frac{x^2 \to 16}{x^2-x-12 \to 0^+} = \infty$



So the 2-sided limit $\boxed{\lim_{x\to 4} \frac{x^2}{x^2-x-12} \text{ DNE.}}$

5. $\lim_{x\to -3} \frac{\frac{1}{3}+\frac{1}{x}}{x+3}$ "type" 0/0. Need algebra.

$= \lim_{x\to -3} \frac{\frac{x+3}{3x}}{x+3}$

$= \lim_{x\to -3} \left(\frac{x+3}{3x}\right)\left(\frac{1}{x+3}\right)$

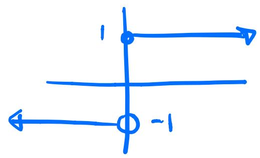
$= \lim_{x\to -3} \frac{1}{3x}$

$= \frac{1}{-9}$

6. Write $\frac{|x|}{x}$ as a piecewise-defined function. $\lim_{x\to 0^-} \frac{|x|}{x} = -1$ $\lim_{x\to 0+} \frac{|x|}{x} = 1$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

So $$\frac{|x|}{x} = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

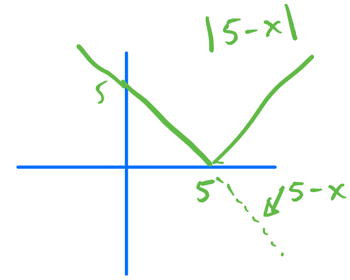


7. $\lim_{x\to 0} \frac{|x|}{x}$ DNE because the one-sided limits do not agree.

8. $\lim_{x\to 5^-} \frac{3x-15}{|5-x|} = \lim_{x\to 5^-} \frac{3(x-5)}{|5-x|} = \lim_{x\to 5^-} \frac{3(x-5)}{5-x}$

$= \lim_{x\to 5^-} \frac{3(x-5)}{-(x-5)}$

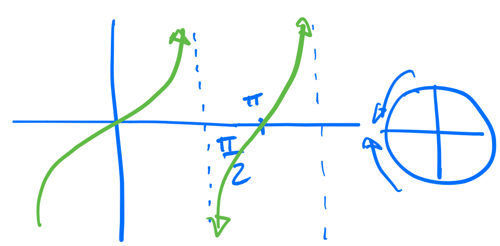
$= -3$



Note if $x < 5$ then $|5-x| = 5-x$

9. $\lim_{x\to\pi} \frac{2x}{\tan^2 x} = \lim_{x\to\pi} \frac{2x}{(\tan(x))^2}$ $2x \to 2\pi$, $(\tan(x))^2 \to 0^+$

$= \infty$



Note as $x\to\pi^-$, $\tan(x)\to 0^-$ and as $x\to\pi^+$, $\tan(x)\to 0^+$. In either case, $(\tan(x))^2 \to 0^+$