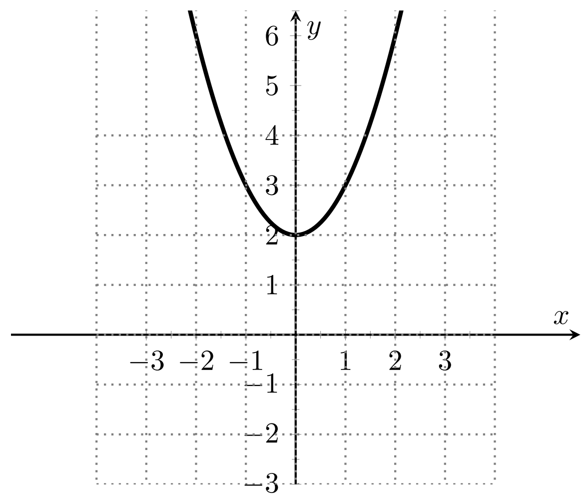
# WORKSHEET §2.7 AND 2.8

The function

$$f'(x) = \underline{\hspace{6cm}}$$

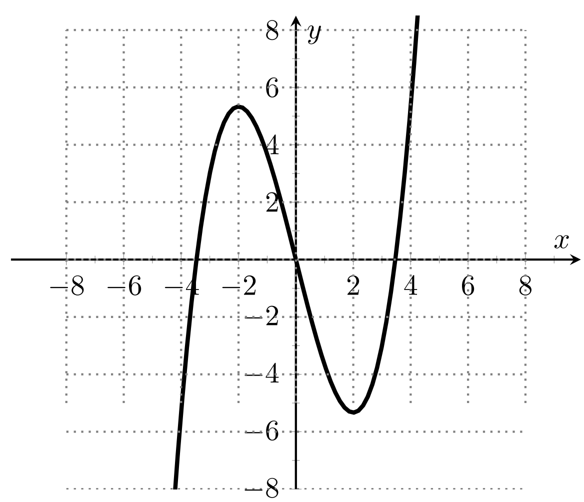
is called the **derivative of** $f$. The value of $f'$ at $x$ can be interpreted geometrically as the ____________ of the tangent line to $f$ at the point $(x, f(x))$. *Note: $f'$ is called the derivative because it has been derived from $f$ using the limit operation defined above. The domain of $f'$ is the set of all $x$ such that this limit exists and may be smaller than the domain of $f$.*

1. Let $f(x) = x^2 + 2$, shown below. Use the definition of the derivative as a function to compute $f'(x)$. Then graph $f'(x)$ on the same axes.




2. Let $f(x) = \frac{1}{3}x^3 - 4x$.

   (a) Use the definition of the derivative (as a function) to find a formula for $f'(x)$. You may find it helpful to use the fact that $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.




   (b) Factor the formula and use the factorization to plot the graph of $f'(x)$ on the same axes that show $f(x)$.

   (c) What do you notice about the relationship between the zeroes of $f'(x)$ and the tangent lines to $f(x)$?

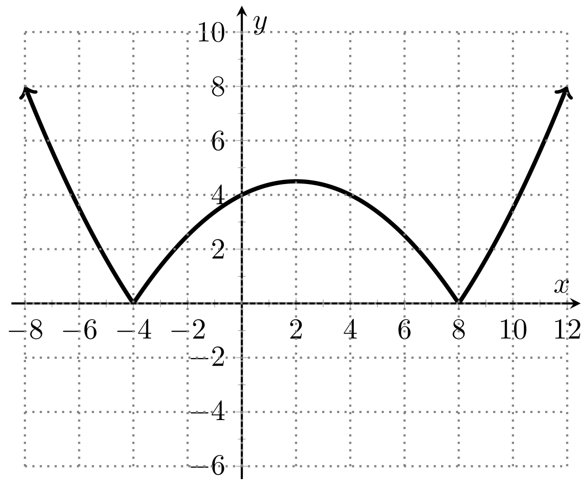
3. Consider the function

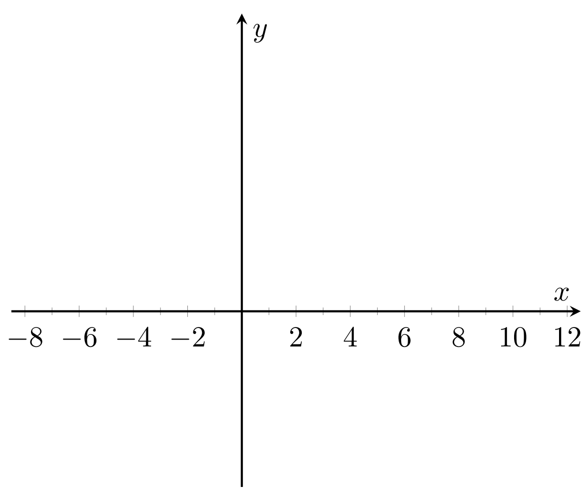
$$f(x) = \left|\frac{x^2}{8} - \frac{x}{2} - 4\right| = \begin{cases} \frac{x^2}{8} - \frac{x}{2} - 4 & \text{if} \quad x \leq -4 \text{ or } x \geq 8 \\ -(\frac{x^2}{8} - \frac{x}{2} - 4) & \text{if} \quad -4 < x < 8 \end{cases}.$$

(a) The graph of $f(x)$ is given on the top set of axes shown below. By thinking about slopes of tangent lines, sketch a graph of the derivative on the second set of axes.

When I ask you to sketch, I am interested in the qualitative behavior of the derivative: Where does it cross the $x$-axis? Is it positive or negative? Is it a lot positive or a little positive? Are the slopes growing steeper or getting less steep? (This is why the $y$-axis is unmarked on the answer graph.)

(b) Use the definition of the derivative to determine $f'(x)$ algebraically, for two cases: (i) $x < -4$ or $x > 8$; (ii) $-4 < x < 8$. Explain why your algebraic calculations match your sketch.




(c) Using your formula from (a), compute

- $\lim_{x\to -4^-} f'(x) =$
- $\lim_{x\to -4^+} f'(x) =$
- $\lim_{x\to 8^-} f'(x) =$
- $\lim_{x\to 8^+} f'(x) =$

Using the language of calculus, what can you say about $f'(x)$ at $x = -4$ and $x = 8$? Why does this make sense geometrically? (Does it match your picture?)