Section 3.2: More Product Rule and Quotient Rule
Intro to Section 3.3

1. For each of the following, compute the derivative using the product or quotient rule, and then compute the derivative without using the product or quotient rule, after the indicated algebraic pre-processing. Do you get the same answer? Which one is easier?

(a) $F(x) = \dfrac{e^x + 1}{\sqrt{2}+1} = \dfrac{1}{\sqrt{2}+1}(e^x + 1)$

with product/quotient

It's crazy to try to use quotient rule here! $\sqrt{2}+1$ is just a constant. But if you do...

$F'(x) = \dfrac{(\sqrt{2}+1)(e^x) - (e^x+1)(0)}{(\sqrt{2}+1)^2} = \dfrac{e^x}{\sqrt{2}+1}$

=

without product/quotient

$F'(x) = \dfrac{1}{\sqrt{2}+1} e^x$

(b) $H(s) = \dfrac{A}{B\sqrt{s}} = \dfrac{A}{B} s^{-1/2}$

with product/quotient

Again, quotient is a pain. But here we go:

$H'(s) = \dfrac{(B\sqrt{s})(0) - (A)(B \cdot \frac{1}{2} s^{-1/2})}{(B\sqrt{s})^2}$

$= \left(\dfrac{-A\cancel{B}}{2\sqrt{s}}\right)\left(\dfrac{1}{B^{\cancel{2}} s}\right) = \dfrac{-A}{2s\sqrt{s}} = \dfrac{-A}{2s^{3/2}}$

=

without product/quotient

$H(s) = \dfrac{A}{B} s^{-1/2}$

$H'(s) = \dfrac{A}{B}\left(\dfrac{-1}{2} s^{-3/2}\right)$

$= \dfrac{-A}{2Bs^{3/2}}$

(c) $k(t) = \dfrac{t^3 + 6}{t} = t^2 + 6t^{-1}$

with product/quotient

$k'(t) = \dfrac{t(3t^2) - (t^3+6)(1)}{t^2}$

$= 3t - t - \dfrac{6}{t^2}$

$= 2t - \dfrac{6}{t^2}$

=

without product/quotient

$k'(t) = 2t - 6t^{-2}$

(d) $y = (x - \pi)e^x = xe^x - \pi e^x$

with product/quotient

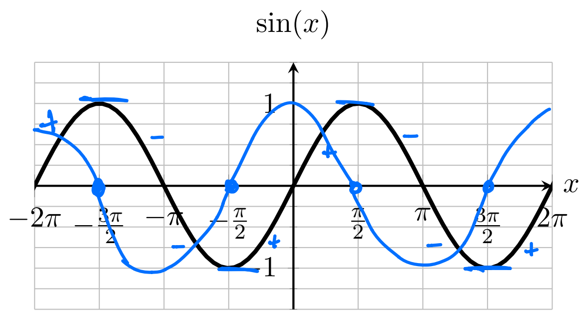
$y' = (x-\pi)e^x + e^x(1)$

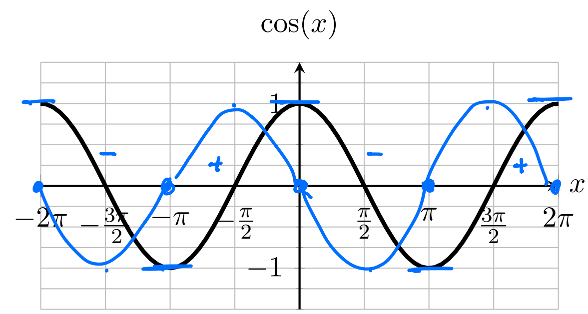
without product/quotient

$y' = xe^x + e^x - \pi e^x$

Actually, these are about the same in terms of difficulty; probably I'd use the product rule!

2. Use the graphs of $y = \sin x$ and $y = \cos x$ to sketch a graph of y' (on the same set of axes).





What do you notice?

looks like $\cos(x)$ looks like $-\sin(x)$

3. Use the fact that $\frac{d}{dx}\sin(x) = \cos(x)$ and $\frac{d}{dx}\cos(x) = -\sin(x)$ to find the derivative of

$$y = 3x^4\cos(x).$$

$$y' = 3x^4 \frac{d}{dx}(\cos(x)) + \cos(x)\frac{d}{dx}(3x^4)$$
$$= 3x^4(-\sin(x)) + \cos(x)(12x^3)$$

4. Use the fact that $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$ and the quotient rule to determine the derivative of $y = \tan(x)$. Simplify your answer as much as possible.

$$\frac{d}{dx}(\tan(x)) = \frac{d}{dx}\left(\frac{\sin(x)}{\cos(x)}\right)$$
$$= \frac{\cos(x)(\cos(x)) - \sin(x)(-\sin(x))}{(\cos(x))^2}$$
$$= \frac{(\cos(x))^2 + (\sin(x))^2}{(\cos(x))^2}$$
$$= \frac{1}{(\cos(x))^2}$$
$$= (\sec(x))^2.$$

So $\boxed{\frac{d}{dx}\tan(x) = (\sec(x))^2.}$