1. Fill in the table below.

Derivatives of Trigonometric Functions:

- $\frac{d}{dx}(\sin x) =$ $\cos(x)$
- $\frac{d}{dx}(\cos x) =$ $-\sin(x)$
- $\frac{d}{dx}(\tan x) =$ $(\sec(x))^2$
- $\frac{d}{dx}(\csc x) =$ $-\csc(x)\cot(x)$
- $\frac{d}{dx}(\sec x) =$ $\sec(x)\tan(x)$
- $\frac{d}{dx}(\cot x) =$ $-(\csc(x))^2$

2. Find the derivative of $y = \dfrac{\sec x}{1 - x\tan x}$.

$$y' = \frac{(1-x\tan(x))(\sec(x)\tan(x)) - (\sec(x))(-[x(\sec(x))^2 + \tan(x)])}{(1-x\tan(x))^2}$$

$$= \frac{(1-x\tan(x))(\sec(x)\tan(x)) + \sec(x)(x(\sec(x))^2 + \tan(x))}{(1-x\tan(x))^2}$$

3. If $f(\theta) = e^\theta \sin(\theta)$, find $f''(\theta)$. Simplify your answers.

$$f'(\theta) = e^\theta\cos(\theta) + \sin(\theta)e^\theta = e^\theta(\cos\theta + \sin\theta)$$

$$f''(\theta) = e^\theta(-\sin\theta + \cos\theta) + (\cos(\theta) + \sin\theta)e^\theta$$

$$= -e^\theta\sin\theta + e^\theta\cos\theta + e^\theta\cos\theta + e^\theta\sin\theta$$

$$= 2e^\theta\cos\theta$$

4. Find $\dfrac{d}{dt}[t\sin t\cos t]$.

$$\frac{d}{dt}((t\sin t)\cos t) = (t\sin(t))(-\sin t) + \cos(t)\frac{d}{dt}(t\sin t)$$

$$= -t(\sin(t))^2 + \cos(t)[t\cos(t) + \sin(t)]$$

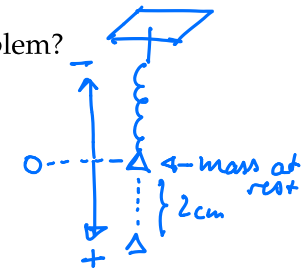
$$= -t(\sin(t))^2 + t(\cos(t))^2 + \sin(t)\cos(t) \quad \text{(just for fun)}$$

5. An elastic band is hung on a hook and a mass is hung on the lower end of the band. When the mass is pulled down 2 cm past its rest position and then released, it vibrates vertically. The equation of motion is

$$s = 2\cos t + 3\sin t, \quad \text{for } t \geq 0,$$

where s is measured in centimeters and t is measured in seconds. (We are taking the positive direction to be downward.)

(a) Why might you expect to use sines and cosines to model this particular problem?



The bouncing is periodic, as are sines & cosines

(b) Find $s(0)$, $s'(0)$, and $s''(0)$ including units.

$s'(t) = 2(-\sin t) + 3\cos(t) = -2\sin t + 3\cos t$

$s''(t) = -2\cos t - 3\sin t$

$s(0) = 2$ cm

$s'(0) = 3$ cm/s

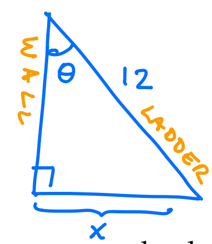
$s''(0) = -2$ cm/s/s $= -2$ cm/s^2

(c) What do the numbers from part (a) indicate about the mass in the context of the problem?

- The displacement at time $t=0$ is 2 cm
- The initial velocity is 3 cm/s
- The initial acceleration is -2 cm/s^2

6. A 12 foot ladder rests against a wall. Let θ be the angle between the ladder and the wall and let x be the distance from the base of the ladder and the wall.

(a) Compute x as a function of θ. (Drawing a picture will help.)

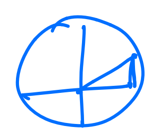


$\frac{x}{12} = \frac{\text{opp}}{\text{hyp}} = \sin\theta \Rightarrow x = 12\sin\theta$

(b) How fast does x change with respect to θ when $\theta = \pi/6$? (Get an exact answer and a decimal approximation.)

$\frac{dx}{d\theta} = 12\cos\theta$ so $\left.\frac{dx}{d\theta}\right|_{\theta=\pi/6} = 12\cos(\pi/6) = \frac{12\cdot\sqrt{3}}{2} = 6\sqrt{3}$

≈ 10.39



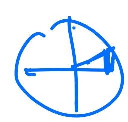
(c) Interpret your answer from part (b) in the context of the problem. (Units will help you here.)

When the angle is $\pi/6$, the end of the ladder is sliding away from the wall at a rate of $6\sqrt{3}$ ft/radians ≈ 10.39 ft/radians

(d) Determine how far the ladder is from the wall when $\theta = \pi/6$.

When $\theta = \pi/6$, $x = 12\sin(\pi/6) = 12\cdot\frac{1}{2} = 6$

The ladder is 6 ft from the wall.



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