SECTION 3.4 CHAIN RULE

1. Complete the Chain Rule (using both types of notation)

- If $F(x) = f(g(x))$,

  then $F'(x) =$ $f'(g(x)) \cdot g'(x)$

- If $y = f(u)$ and $u = g(x)$,

  then $\frac{dy}{dx} =$ $\frac{dy}{du} \cdot \frac{du}{dx}$

Derivative of outside with respect to inside, times derivative of the inside.

2. For each function below, write it as a nontrivial composition of functions in the form $f(g(x))$. Then use the chain rule to compute the derivative.

(a) $H(x) = \sqrt[3]{4 - 2x}$

$f(x) =$ x^3

$g(x) =$ $4-2x$

$H'(x) = 3(g(x))^2 \cdot g'(x)$

$= 3(4-2x)^2 (-2)$

(b) $H(x) = \tan(2 - x^4)$

$f(x) =$ $\tan(x)$

$g(x) =$ $2-x^4$

$H'(x) = (\sec(2-x^4))^2 (-4x^3)$

(c) $H(x) = e^{2-2x^3}$

$f(x) =$ e^x

$g(x) =$ $2-2x^3$

$H'(x) = (e^{2-2x^3})(-6x^2)$

(d) $H(x) = \frac{4}{x+\sin(x)}$ $= 4(x+\sin(x))^{-1}$

$f(x) =$ $4x^{-1}$

$g(x) =$ $x+\sin(x)$

$H'(x) = 4(-1)(x+\sin(x))^{-2}(1+\cos(x))$

3. For each problem below, find the derivative.

(a) $z(t) = (2x^3 - 5x)^7$

$z'(t) = 7(2x^3 - 5x)^6(6x^2 - 5)$

(b) $x(\theta) = (\cos(\theta))^3$

$x'(\theta) = 3(\cos(\theta))^2(-\sin\theta)$

(c) $y = x^2 - 3\sin(x^3)$

$y' = 2x - 3\cos(x^3)(3x^2)$

(d) $y = 10e^{\sqrt{t}}$

$y' = 10\,e^{\sqrt{t}}\left(\frac{1}{2}t^{-1/2}\right)$

(e) $f(x) = \dfrac{\sqrt{2}}{\sqrt{x^2 - 4}}$ $= \sqrt{2}\,(x^2-4)^{-1/2}$

$f'(x) = \sqrt{2}\left(-\frac{1}{2}(x^2-4)^{-3/2}\right)(2x)$

(f) $g(x) = \dfrac{\sec(x^2 + 2)}{12}$ $= \frac{1}{12}\sec(x^2+2)$

$g'(x) = \frac{1}{12}\sec(x^2+2)\tan(x^2+2)(2x)$

(g) $k(s) = \dfrac{A^2}{B + Cs}$ (A, B, C are constants!) $= A^2(B+Cs)^{-1}$

$k'(s) = A^2(-1)(B+Cs)^{-2}(C)$

$= \dfrac{-A^2C}{B+Cs}$

Compare: $k'(s) = \dfrac{(B+Cs)(0) - A^2C}{(B+Cs)^2}$

$= \dfrac{-A^2C}{B+Cs}$ as well