SECTION 3.6 DERIVATIVES OF LOGARITHMIC FUNCTIONS

1. Fill in the derivative rules below:

$$\frac{d}{dx}[\arcsin(x)] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arccos(x)] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arctan(x)] = \frac{1}{1+x^2}$$

$$\frac{d}{dx}[\log_b(x)] = \frac{1}{x\ln(b)}$$

$$\frac{d}{dx}[\ln(x)] = \frac{1}{x}$$

2. Find the derivative of each function below:

(a) $y = \ln(x^5)$

$$y' = \frac{1}{x^5}(5x^4)$$

(b) $y = (\ln x)^5$

$$y' = \frac{5(\ln(x))^4}{x}$$

(c) $y = \ln(5x)$

$$y' = \frac{1}{5x}(5) = \frac{1}{x}$$

3. Find the derivative of each function below:

(a) $f(x) = x^2 \log_2(5x^3 + x)$

$$f'(x) = x^2\left[\frac{1}{(5x^3+x)\ln(2)}(15x^2+1)\right] + \log_2(5x^3+x)(2x)$$

(b) $g(x) = \ln(x^2 \tan^2 x)$

$$g'(x) = \frac{1}{x^2(\tan(x))^2} \cdot \frac{d}{dx}\left(x^2 \cdot (\tan(x))^2\right)$$

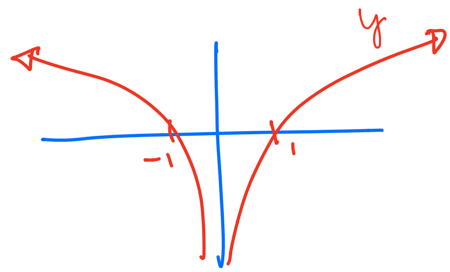
$$= \frac{1}{x^2(\tan(x))^2}\left(x^2 \frac{d}{dx}((\tan(x))^2) + (\tan(x))^2 \frac{d}{dx}(x^2)\right)$$

$$= \frac{1}{x^2(\tan(x))^2}\left(x^2 \cdot 2\tan(x) \cdot \sec(x)\tan(x) + (\tan(x))^2(2x)\right)$$

4. Find $\frac{dy}{dx}$ for $y = \ln \sqrt{\frac{x+\sin x}{x^2-e^x}}$. $= \ln\left(\left(\frac{x+\sin(x)}{x^2-e^x}\right)^{1/2}\right)$ So many compositions

$$\frac{dy}{dx} = \frac{1}{\sqrt{\frac{x+\sin(x)}{x^2-e^x}}} \cdot \frac{1}{2}\left(\frac{x+\sin(x)}{x^2-e^x}\right)^{-1/2}\left(\frac{(x^2-e^x)(1+\cos(x)) - (x+\sin(x))(2x-e^x)}{(x^2-e^x)^2}\right)$$

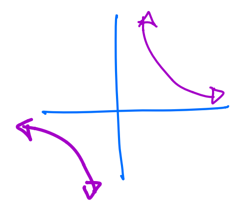
5. Find y' for each of the following:



(a) $y = \ln|x|$

$$y = \begin{cases} \ln(x) & x>0 \\ \ln(-x) & x<0 \end{cases}$$ undefined at 0

$$\text{So } y' = \begin{cases} \frac{1}{x} & x>0 \\ \frac{1}{-x}(-1) & x<0 \end{cases} = \frac{1}{x}, \ (x \neq 0)$$



(b) $y = \frac{e^{-x}\sin x}{\sqrt{1-x^2}}$ ← logarithmic differentiation makes this easier!

Using logarithmic differentiation:

$$\ln(y) = \ln\left(\frac{e^{-x}\sin x}{\sqrt{1-x^2}}\right) = \ln(e^{-x}\sin(x)) - \ln((1-x^2)^{1/2})$$

$$= \ln(e^{-x}) + \ln(\sin(x)) - \tfrac{1}{2}\ln(1-x^2) = -x + \ln(\sin(x)) - \tfrac{1}{2}\ln(1-x^2)$$

$$\frac{1}{y}y' = -1 + \frac{\cos(x)}{\sin x} - \frac{1}{2}\,\frac{-2x}{1-x^2} \implies$$

$$y' = \left(-1 + \frac{\cos(x)}{\sin(x)} - \frac{x}{1-x^2}\right)\left(\frac{e^{-x}\sin(x)}{\sqrt{1-x^2}}\right)$$

(c) $y = x^{\sqrt[3]{x}}$ ← logarithmic differentiation is mandatory

$$\ln(y) = \ln(x^{\sqrt[3]{x}}) = x^{1/3} \cdot \ln(x)$$

$$\frac{y'}{y} = x^{1/3} \cdot \frac{1}{x} + \ln(x) \cdot \frac{1}{3}x^{-2/3} \implies$$

$$y' = \left(x^{1/3} \cdot \frac{1}{x} + \ln(x) \cdot \frac{1}{3}x^{-2/3}\right)\left(x^{\sqrt[3]{x}}\right)$$