1. A particle moves according to the law of motion $s(t) = 2 - 15t + 4t^2 - \frac{1}{3}t^3$, for $t \geq 0$, where t is measured in seconds and s is measured in feet.

   (a) Find the velocity at time t.

   $v(t) = s'(t) = -15 + 8t - t^2$

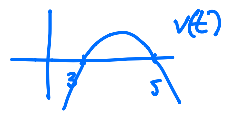
   (b) What is the velocity after 1 second?

   $v(1) = -15 + 8 - 1 = -16 + 8 = -8$ ← at $t = 1$, velocity $= -8$ ft/s

   (c) When is the particle at rest?

   Need $v(t) = 0 \Rightarrow -15 + 8t - t^2 = 0 \Rightarrow t^2 - 8t + 15 = 0 \Rightarrow (t-5)(t-3)$

   ⇒ At rest at $t = 3$ s & $t = 5$ s.

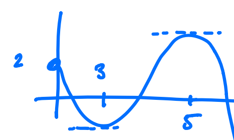


   (d) When is the particle moving in the positive direction?

   $s(0) = 2$

   $s(1) = 2 - 15 + 4 - 1/3 < 0$

   from $t = 3$ to $t = 5$ (where slopes are positive)



   Not to scale, obviously

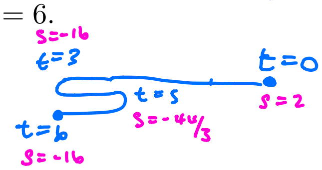
   (e) Draw a diagram of the particle from $t = 0$ to $t = 6$.

   $s(0) = 2$

   $s(3) = -16$

   $s(5) = -\frac{44}{3} = -14\,2/3$

   $s(6) = -16$

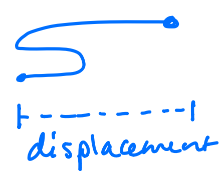


   (f) Find the *displacement* of the particle during the first 6 seconds.

   displacement $= s(6) - s(0) =$

   $[2 - 15(6) + 4(36) - \frac{1}{3}(36 \cdot 6)] - 2 = -15(6) + 4(36) - 2(36)$

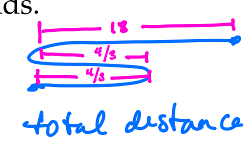
   $= -3 \cdot 5 \cdot 6 + 4 \cdot 6 \cdot 6 - 2 \cdot 6 \cdot 6 = 6(-15 + 24 - 12) = 6(-27 + 24) = -18$



   (g) Find the *total distance traveled* by the particle during the first 6 seconds.

   Note $|s(3) - s(5)| = 16 - \frac{44}{3} = \frac{48}{3} - \frac{44}{3} = \frac{4}{3}$

   So total distance $= 18 + 2(\frac{4}{3}) = 18 + 8/3 = 20\,2/3$



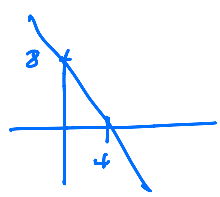
   (h) Find the acceleration of the particle.

   $a(t) = v'(t) = 8 - 2t$

   (i) Graph the acceleration function.

   Check: $8 - 2t = 0 \Rightarrow -2t = -8$

   $\Rightarrow t = 4$



   (j) When is the particle speeding up?

   That is, where is the velocity increasing: $(-\infty, 4)$

   (or $(0, 4)$.)

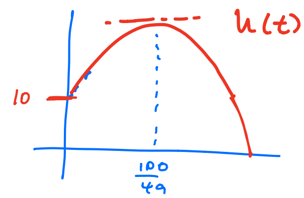
2. The height (in meters) of a projectile shot vertically upward from a point 10 meters above ground lever with an initial velocity of 20 meters per second is $h = 10 + 20t - 4.9t^2$.

   (a) When does the projectile reach its maximum height?

   Max height when $v(t) = 0$

   $v(t) = 20 - 4.9(2t) = 0 \Rightarrow$

   $t = \frac{20}{4.9(2)} = \frac{10}{4.9} = \frac{10}{\frac{49}{10}} = \frac{100}{49}$



   (b) What is its maximum height?

   $h\left(\frac{100}{49}\right) = 10 + 20\left(\frac{100}{49}\right) - 4.9\left(\frac{100}{49}\right)^2$

   $= 30.408$ meters

   (c) When does the projectile hit the ground?

   $h(t) = 0 \Rightarrow 10 + 20t - 4.9t^2 = 0 \Rightarrow t = \frac{-20 \pm \sqrt{20^2 - 4(10)(-4.9)}}{2(-4.9)} = \frac{-20 \pm \sqrt{4(100) - 4(-49)}}{2(-4.9)}$

   $= \frac{10 \pm \sqrt{149}}{4.9} \Rightarrow t = -0.450$ or $t = 4.532$ Only the positive value makes sense! Choose $\boxed{t = 4.532}$

   (d) With ~~What~~ what velocity does it hit the ground?

   Compute $v(4.532) = 20 - 4.9(2(4.532)) = -24.413$ m/s

3. A tank holds 1000 gallons of a fluid, which drains from the bottom of the tank in 30 minutes. The function below give the volume of fluid remaining in the tank after t minutes:

$$V(t) = 1000\left(1 - \frac{1}{30}t\right)^2 \text{ for } 0 \leq y \leq 30$$

   Find the rate at which the fluid is draining from the tank after 10 minutes. When is the fluid flowing the fastest? Slowest?

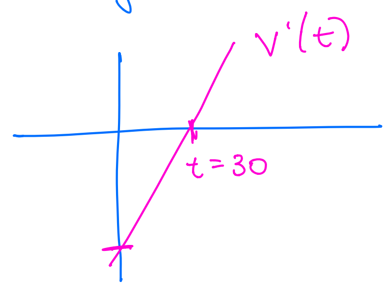
   $V'(t) = 1000(2)\left(1 - \frac{1}{30}t\right)\left(-\frac{1}{30}\right) = \frac{-200}{3}\left(1 - \frac{t}{30}\right)$ ← this is a line!

   $V'(10) = -\frac{200}{3}\left(1 - \frac{10}{30}\right) = -\frac{200}{3}\left(\frac{2}{3}\right) = \frac{-400}{9} = -44.44$ gal/min.

   → Want max & min for $|V'(t)|$

   Max is $|V'(0)| = \frac{200}{3}$ m³/min

   Min is $|V'(30)| = 0$ m³/min



   $V'(t) = \frac{-200}{3} + \frac{20}{9}t$