1. A population of flies is assumed to grow with constant relative growth rate. Suppose there are 100 flies after the second day and 400 flies after the fourth day.

   (a) What is the relative growth rate?

   This means $\frac{dP}{dt} = kP \Rightarrow P(t) = P_0 e^{kt}$

   $P(2) = 100 = P_0 e^{k(2)}$

   $P(4) = 400 = P_0 e^{4k} \Rightarrow$

   $\frac{400}{100} = \frac{P_0 e^{4k}}{P_0 e^{2k}} \Rightarrow 4 = e^{k(4-2)} \Rightarrow \ln(4) = 2k$

   $\Rightarrow k = \frac{\ln(4)}{2} = \frac{\ln(2^2)}{2} = \frac{2\ln(2)}{2} = \boxed{\ln(2)}$

   $\rightarrow k = \ln(2)$ so $P(t) = P_0 e^{\ln(2) t} = P_0 \cdot 2^t$

   (b) What was the initial size of the population?

   $P(t) = P_0 e^{\ln(2) t} \Rightarrow P(2) = 100 = P_0 \cdot 2^2 \Rightarrow P_0 = 25$

   ← plausible?

t	#
0	25
1	50
2	100
3	200
4	400
5	800
6	1600
7	3200
8	6400
9	12800

   Yes!

   (c) Find an expression for the number of flies after ~~t~~ d days.

   $P(t) = 25 \cdot 2^t$

   (d) When will the population of flies be 10,000?

   $P(t) = 10000 \Rightarrow 25 \cdot 2^t = 10000 \Rightarrow 2^t = \frac{100(100)}{25} = 400$

   $\Rightarrow t = \log_2(400) = \frac{\ln(400)}{\ln(2)} \approx 8.6$

   plausible!

2. If A_0 dollars are invested at an annual interest rate r, but interest is compounded n times per year, then the total amount of money after t years is

$$V(t) = A_0 \left(1 + \frac{r}{n}\right)^{nt}.$$

   (a) Suppose \$1000 is invested at a rate of 2.5% for 10 years. How much money do you have if interest is compounded

   i. annually

   $V(1000) = 1000(1.025)^{10} = 1280.08$

   ii. quarterly

   $1000\left(1 + \frac{0.025}{4}\right)^{4 \cdot 10} = 1283.03$

   iii. monthly

   $1000\left(1 + \frac{0.025}{12}\right)^{12 \cdot 10} = 1283.69$

   $n = \frac{n}{r} \cdot r$

   (b) If we let $n \to \infty$, then we say that interest is compounded *continuously*. Using the fact that

$$e = \lim_{n\to\infty} \left(1 + \frac{1}{n/r}\right)^{n/r},$$

   $\lim_{n\to\infty} A_0\left(1 + \frac{r}{n}\right)^{nt} = A_0 \lim_{n\to\infty}\left(\left(1 + \frac{1}{n/r}\right)^{n/r}\right)^{rt}$ ← constants wrt n; use continuity here!

   $= A_0\left(\lim_{n\to\infty}\left(1 + \frac{1}{n/r}\right)^{n/r}\right)^{rt}$

   $= A_0 e^{rt}$

   and letting $m = n/r$, determine how much money you have after 10 years if interest is compounded continuously.

   $V(t) = 1000 e^{rt} = 1284.02$

3. Radium-226 has a half-life of 1590 years. Assume one starts with 50 mg of Radium-226. Find an expression for the amount m of Radium-226 in terms of t. How much radium is left after

$m(0) = 50 = M_0$

$m(1590) = 25$

$M(t) = 50e^{kt} \Rightarrow 25 = 50e^{1590t} \Rightarrow \frac{1}{2} = e^{1590t} \Rightarrow$

$\ln(1/2) = 1590t$

So $m(t) = 50\left(e^{\frac{\ln(1/2)}{1590}}\right)^t$ $\qquad k = \frac{\ln(1/2)}{1590}$

4. When a cup of coffee is poured it is 95 degrees Celsius. After 10 minutes the coffee has cooled to 80 degrees Celsius. If the surrounding temperature is 20 degrees Celsius, find an expression for the temperature of the coffee T in terms of time t. You will need to use Newton's Law of Cooling, which says that the rate of cooling is proportional to the difference between the starting temperature and the temperature of the ambient space.

$$\frac{dT}{dt} = k(T - T_{amb}) \Rightarrow \text{Let } y = T - T_{amb}. \text{ Then } \frac{dy}{dt} = \frac{dT}{dt}.$$

So $y = y_0 e^{kt}$ where $y_0 = T_{start} - T_{amb} = 95 - 20 = 75$

and $y(10) = T(10) - T_{amb} = 80 - 20 = 60 = 75e^{10k} \Rightarrow e^{10k} = \frac{60}{75} = \frac{12}{15} = \frac{4}{5}.$

Therefore, $10k = \ln(4/5) \Rightarrow k = \ln(4/5)/10.$ So

$T(t) - T_{amb} = 75e^{(\ln(4/5)/10)t} \Rightarrow T(t) = 75e^{\frac{\ln(4/5)}{10}t} + 20.$