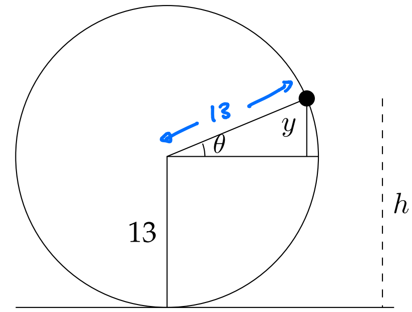
SECTION 3.9: RELATED RATES – DAY 2
SECTION 3.10 TANGENT LINE APPROXIMATION INTRO

1. A Ferris wheel with a radius of 13 meters is rotating at a rate of one revolution every three minutes. How fast is a rider rising when her seat is 18 meters above the ground? (Assume the wheel is tangent to the ground at the bottom.) *Hint: Label useful things in the diagram sketch.*



(a) In terms of the labels given in the picture and calculus-type language:

- What do we KNOW? (Hint: how many radians in one revolution?)

$r = 13 \qquad \frac{d\theta}{dt} = \frac{1 \text{ revolution}}{3 \text{ minutes}} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}} = \frac{2\pi}{3} \text{ rad/min}$

- What do we WANT?

$\frac{dh}{dt}$ when $h = 18$

(b) Determine an equation that relates the variables in your WANT and KNOW.

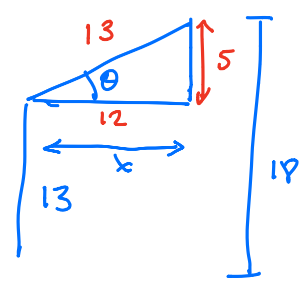
Know $h = 13 + y$ and $\frac{y}{13} = \frac{\text{opp}}{\text{hyp}} = \sin(\theta) \Rightarrow y = 13\sin\theta$

So $h(t) = 13 + 13\sin\theta$

(c) Solve the related rates problem.

(Hint: use what you know about right-triangle trigonometry! You don't actually need to know the angle from horizontal she's at when she's 18 feet above the ground.)

$$\frac{dh}{dt} = 13\cos\theta \, \frac{d\theta}{dt}$$



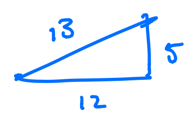
When $h = 18$, $y = 5$, so $x = 12$

and therefore, $\cos\theta = \frac{\text{opp}}{\text{hyp}} = \frac{12}{13}$

Thus

$$\frac{dh}{dt} = \cancel{13}\left(\frac{\overset{4}{\cancel{12}}}{\cancel{13}}\right)\left(\frac{2\pi}{\cancel{3}}\right)$$

$$= 8\pi \text{ ft/min}$$



is a pythagorean triple; otherwise you can use the pythagorean theorem to find the ⟷ length

2. Consider the function $f(x) = x^3$.

(a) At the point $x = 2$, what is $f(x)$? $f(2) =$ 8

(b) Let $L(x)$ be the function that is the tangent line to $f(x)$ at $x = 2$. This tangent line is sometimes called the *linearization* of $f(x)$ at $x = 2$. Finish the equation (you will need to show some work):

$f'(x) = 3x^2$ so $f'(2) = 3(2)^2 = 12$ ← this is the slope

$L(x)$ passes through $(2, 8)$.

$L(x) =$ $12(x-2)+8$.

(c) Observe that the value $x = 2.1 = 2 + \frac{1}{10}$ is very close to $x = 2$. Evaluate $L(x)$ at $x = 2.1$. Do not use a calculator until your very last step (that is you can get a decimal approximation of a fraction, but you should compute the fraction by hand).

$L(2+\frac{1}{10}) = 12\left(2+\frac{1}{10}-2\right)+8 = \frac{12}{10} + \frac{80}{10} = \frac{92}{10} = \frac{46}{5} = 45 + \frac{1}{5}$

$L(2.1)$ as a fraction $\frac{46}{5}$ $L(2.1)$ as a decimal approximation. 9.2

(d) Use a calculator or a computer to evaluate $f(2.1)$. $f(2.1) =$ $(2.1)^3 = 9.261$

(e) What is the error if you use $L(2.1)$ to approximate $f(2.1)$? (That is, what is the difference between the two quantities?) What is the percent error, calculated as (approx value - actual value)/(actual value)?

error $= 9.261 - 9.2 = 0.061$

% error $= \frac{(0.061)}{9.261} = 6.5\%$

(f) Draw a rough sketch of $f(x)$ and $L(x)$, and use the picture and your computations to explain, in a sentence or two, why using $L(2.1)$ to approximate the cube of 2.1 is a reasonable thing to do.

The distance between $f(x)$ & $L(x)$ is very small when x is very close to 2 — we can see that in the picture and we computed that above.