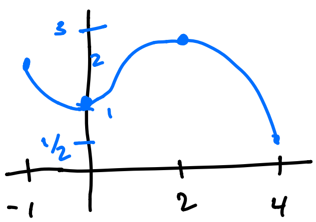
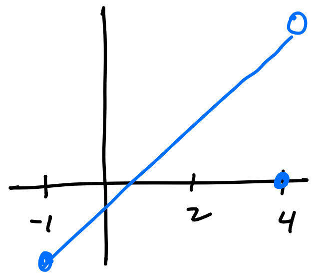
1. Sketch ~~a~~ graphs of 3 functions $f(x)$ whose domain is the interval $[-1, 4]$ with the following properties:

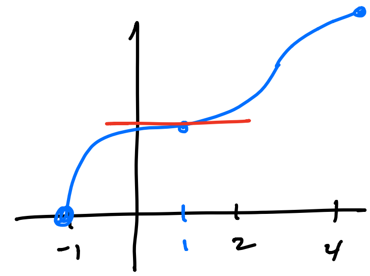
(a) f is continuous, has a local minimum at $x = 0$, an absolute minimum at $x = 4$ and an absolute maximum at $x = 2$.



(b) f has an absolute minimum but no absolute maximum



(c) f has a critical point at $x = 1$ but no maximum or minimum (of any type) at $x = 1$.



2. Find the absolute maximum and minimum values of $f(x) = x - x^{1/3}$ on the interval $[-1, 4]$. Determine where those absolute maximum and minimum values occur.

Need to check: 1) endpoints

2) Critical points: where f' is undefined and where $f' = 0$.

Critical points: $f'(x) = 1 - \frac{1}{3}x^{-2/3} = 1 - \frac{1}{3\sqrt[3]{x^2}}$.

f' is undefined at $x = 0$.

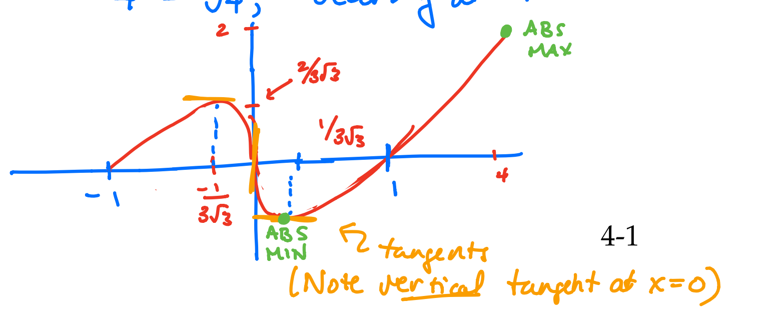
$f' = 0$ where $1 = \frac{1}{3\sqrt[3]{x^2}} \Rightarrow x^{2/3} = 1/3 \Rightarrow x^2 = \frac{1}{27} \Rightarrow x = \frac{-1}{3\sqrt{3}}$ or $x = \frac{1}{3\sqrt{3}}$.

≈ -0.192 $\quad \approx 0.192$

type	x	$f(x)$
endpt	-1	0
crit. pt	$\frac{-1}{3\sqrt{3}}$	$\frac{2}{3\sqrt{3}} \approx 0.38$
crit pt	0	0
crit pt	$\frac{1}{3\sqrt{3}}$	$\frac{-2}{3\sqrt{3}} \approx -0.38$ ← ABS MIN
endpt	4	$4 - \sqrt[3]{4} \approx 2.416$ ← ABS MAX

Answer

- f has an absolute min. value of $\frac{-2}{3\sqrt{3}}$ occuring at $x = \frac{1}{3\sqrt{3}}$
- f has an absolute maximum value of $4 - \sqrt[3]{4}$, occuring at $x = 4$.



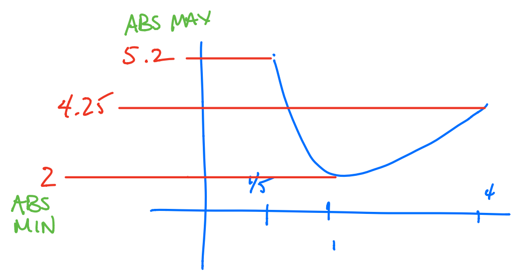
3. Find the absolute maximum and minimum values of $f(x) = x + \dfrac{1}{x}$ on the interval $[1/5, 4]$. Determine where those absolute maximum and minimum values occur.

→ Check endpoints: $x = 1/5$ & $x = 4$. (Note f has VA at $x=0$ which is outside our domain)

→ Check critical points. ⇒ where $f'(x) = 0$ and where $f'(x)$ is undefined.

$f'(x) = 1 - \frac{1}{x^2}$. So f' is undefined at $x=0$ (not in our domain) and $f'(x) = 0 \Rightarrow 0 = 1 - \frac{1}{x^2} \Rightarrow \frac{1}{x^2} = 1 \Rightarrow x^2 = 1 \Rightarrow x = 1$ or $x = -1$. Only $x=1$ is in our domain.

x	$f(x)$
$1/5$	$5 + 1/5 = \frac{26}{5} = 5.2$ ← Absolute max, at $x = 1/5$
1	$1 + 1 = 2 = 2$ ← Absolute Min, at $x = 1$
4	$\frac{1}{4} + 4 = \frac{17}{4} = 4.25$



4. Find the absolute maximum and minimum values of $f(x) = x^{2/3}$ on the interval $[-8, 8]$. Determine where those absolute maximum and minimum values occur.

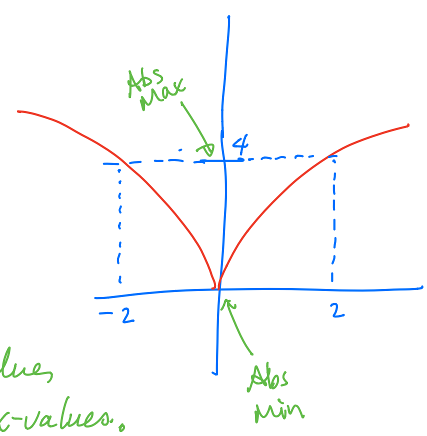
Endpoints: $x = -8$ and $x = 8$

Critical points: $f'(x) = \frac{2}{3} x^{-4/3} = \dfrac{2}{3\sqrt[3]{x^4}}$

- $f'(x)$ is undefined at $x = 0$
- $f'(x) = 0$ never, because $\dfrac{2}{3\sqrt[3]{x^4}} = 0 \Rightarrow 2 = 0$ which is not true.

x	$f(x)$
-8	$(-8)^{2/3} = (-2)^2 = 4$ ← Abs Max
0	0 ← Abs Min, at $x=0$
8	$8^{2/3} = 2^2 = 4$ ← Abs Max

Abs Max at $x = 8$ & $x = -8$



Notice there is only one absolute maximum value, $y = 4$, but it is attained at two different x-values.
There is only one absolute minimum value, $y = 0$, attained at $x = 0$ (where it turns out the function has a cusp.)