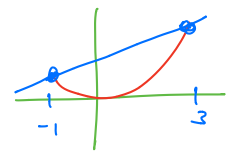
1. Write down the statement of the Mean Value Theorem.

If f is continuous on $[a,b]$ and differentiable on (a,b) then there exists some c in (a,b) such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$. (That is, some TL is parallel to the secant line connecting the endpoints)

2. Consider the function $f(x) = x^2$ on the interval $[-1, 3]$

   (a) Find the slope of the secant line of the graph of $f(x)$ from $x = -1$ to $x = 3$.

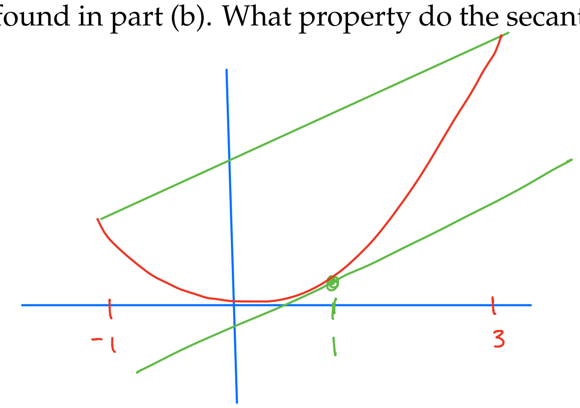
   Secant slope $= \dfrac{3^2 - (-1)^2}{3-(-1)} = \dfrac{9-1}{3+1} = \dfrac{8}{4} = 2$



   (b) Find a value of x in $[-1, 3]$ where $f'(x)$ equals the value in part a.

   Solve $f'(x) = 2 \Rightarrow 2x = 2 \Rightarrow x = 1$

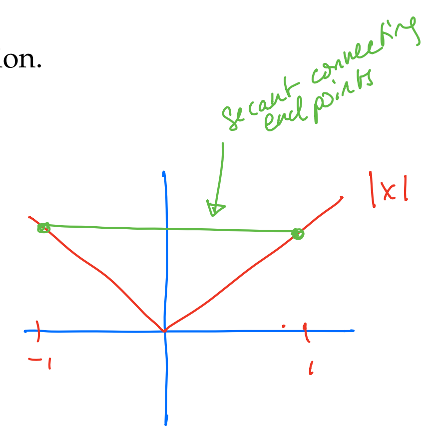
   (c) Make a sketch of the graph of $f(x)$ and add to it the secant line from part (a) and the tangent line at the location found in part (b). What property do the secant line and tangent line have?



3. Consider the function $f(x) = |x|$ on $[-1, 1]$. Sketch a graph of this function.

   (a) What would MVT say about f on $[-1, 1]$?

   It would say there is some point c so that the TL to $f(x)$ at c has slope 0, because
   $$\frac{|1| - |-1|}{1-(-1)} = \frac{1-1}{1+1} = \frac{0}{2} = 0$$



   (b) Does MVT "work" in this case? Why or why not?

   No! The hypotheses are not satisfied: $f(x)$ is <u>not</u> differentiable on $(-1,1)$

4. Suppose a car is traveling down the road and in 30 minutes it travels 32.7 miles. What does the Mean Value Theorem have to say about this?

Average speed is $\dfrac{32.7 \text{ miles}}{1/2 \text{ hour}}$ so at some point

it must have had an instantaneous velocity of 16.35 miles/hour

5. Suppose that $f(0) = -3$ and that $f'(x)$ exists and is less than or equal to 5 for all values of x. How large can $f(2)$ possibly be?

MVT says for some c, $f'(c) = \dfrac{f(b)-f(a)}{b-a} = \dfrac{f(2)-f(0)}{2-0}$ but

$f'(c) \leq 5$, so $\dfrac{f(2)-(-3)}{2} = f'(c) \leq 5 \Rightarrow f(2)+3 \leq 10 \Rightarrow f(2) \leq -7.$

6. (a) Suppose $f(x) = C$ on $[a, b]$, where C is a fixed constant. What can you say about $f'(x)$?

$f'(x) = 0.$

(b) Suppose $f(x)$ is continuous on $[a, b]$ and $f'(x) = 0$ on (a, b). Choose any x in (a, b). Because we know $f'(x) = 0$ on (x, b), MVT says there exists some c in (x, b) such that

$$\frac{f(b) - f(x)}{b - a} = f'(c) = \underline{\ 0\ }. \Rightarrow f(b) - f(x) = 0(b-a) = 0$$

What can you say about $f(x)$?

$f(x) = f(b)$ (that is, $f(x)$ = constant)

(c) Now suppose that f and g are continuous and $f'(x) = g'(x)$ for all x in the interval (a, b).

i. Let $H(x) = f(x) - g(x)$. What can you say about $H'(x)$?

$H'(x) = f'(x) - g'(x) = 0$ since $f'(x) = g'(x)$

ii. What can you conclude about the relationship between $f(x)$ and $g(x)$?

$f(x) = g(x)$ + some constant!

because $H(x)$ = some constant