SECTION 4.2: THE MEAN VALUE THEOREM

1. Write down the statement of the Mean Value Theorem.

2. Consider the function $f(x) = x^2$ on the interval $[-1, 3]$

   (a) Find the slope of the secant line of the graph of $f(x)$ from $x = -1$ to $x = 3$.

   (b) Find a value of x in $[-1, 3]$ where $f'(x)$ equals the value in part a.

   (c) Make a sketch of the graph of $f(x)$ and add to it the secant line from part (a) and the tangent line at the location found in part (b). What property do the secant line and tangent line have?

3. Consider the function $f(x) = |x|$ on $[-1, 1]$. Sketch a graph of this function.

   (a) What would MVT say about f on $[-1, 1]$?

   (b) Does MVT "work" in this case? Why or why not?

4. Suppose a car is traveling down the road and in 30 minutes it travels 32.7 miles. What does the Mean Value Theorem have to say about this?

5. Suppose that $f(0) = -3$ and that $f'(x)$ exists and is less than or equal to 5 for all values of x. How large can $f(2)$ possibly be?

6. (a) Suppose $f(x) = C$ on $[a, b]$, where C is a fixed constant. What can you say about $f'(x)$?

   (b) Suppose $f(x)$ is continuous on $[a, b]$ and $f'(x) = 0$ on (a, b). Choose any x in (a, b). Because we know $f'(x) = 0$ on (x, b), MVT says there exists some c in (x, b) such that

   $$\frac{f(b) - f(x)}{b - a} = f'(c) = \_\_\_\_\_.$$

   What can you say about $f(x)$?

   (c) Now suppose that f and g are continuous and $f'(x) = g'(x)$ for all x in the interval (a, b).

   i. Let $H(x) = f(x) - g(x)$. What can you say about $H'(x)$?

   ii. What can you conclude about the relationship between $f(x)$ and $g(x)$?