1. Suppose $f(x) = x^5 - 5x^3$.
Note $f'(x) = 5x^4 - 15x^2 = 5x^2(x^2 - 3)$ and $f''(x) = 20x^3 - 30x = 10x(2x^2 - 3)$.

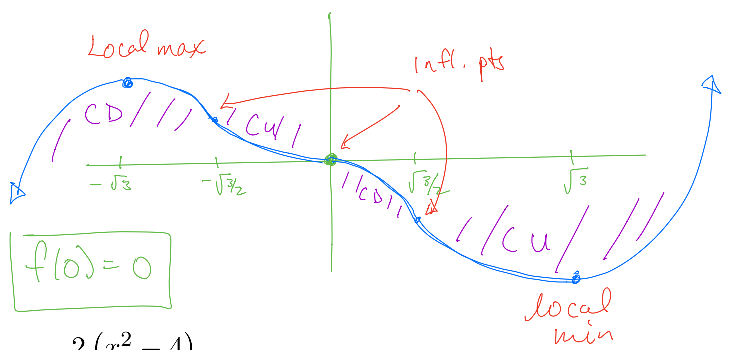
   (a) Find all critical points of $f(x)$.
   (b) Determine the open intervals on which f is increasing or decreasing, and classify each critical point as a local minimum, a local maximum, or neither. (Make a table!)
   (c) Find the intervals of concavity and the inflection points. (Make a table!)
   (d) Put together all this information and sketch the shape of the graph.

- Critical Points: $f'(x) = 0 \Rightarrow x = 0$ or $x^2 = 3 \Rightarrow x = \sqrt{3}$ or $x = -\sqrt{3}$. Note $f'(x)$ is cont's on $\mathbb{R}$ so no other critical points.
- Possible Inflection points: $f''(x) = 0 \Rightarrow x = 0$ or $2x^2 - 3 = 0 \Rightarrow x = \sqrt{\frac{3}{2}}$ or $x = -\sqrt{\frac{3}{2}}$.

Note $4 < 3 < 1$
So $2 < \sqrt{3} < 1$
$1 < 3/2 < 0$ so
$1 < \sqrt{3/2} < 0$

Table:

x	$x < -\sqrt{3}$	$-\sqrt{3}$		$-\sqrt{3/2}$		0		$\sqrt{3/2}$		$\sqrt{3}$	
Sample	-2		$-3/2$		-1		1		$3/2$		2
f'	+	0	−	−	−	0	−	−	−	0	+
Incr/Decr	↗	MAX	↘	↘	↘		↘	↘	↘	MIN	↗
f''	−	−	−	0	+	0	−	0	+	+	+
Concavity	∩	∩	∩	INF	∪	INF	∩	INF	∪	∪	∪



$\boxed{f(0) = 0}$

$f'(x) = 5x^2(x^2 - 3)$
$f'(-2) = 5(-2)^2((-2)^2 - 3) = 5(+)(+)$
$f'(-3/2) = 5(-3/2)^2((-\frac{3}{2})^2 - 3) = 5(+)(\frac{9}{4} - 3) = (+)(-)$
$f'(-1) = 5(-1)^2((-1)^2 - 3) = 5(+)(-)$
$f'(3/2) = 5(3/2)^2((3/2)^2 - 3) = 5(+)(-)$
$f'(2) = 5(2)^2(2^2 - 3) = 5(+)(+)$

→ This was dumb. Only needed to check −1 & 1

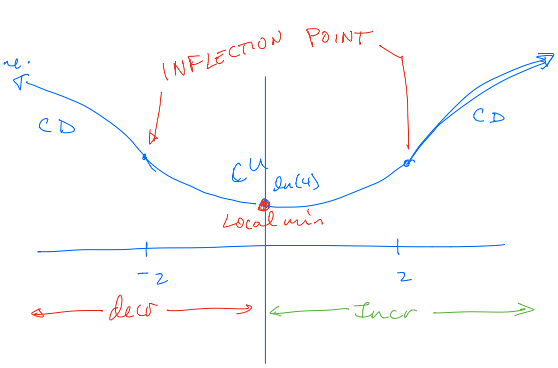
$f''(x) = 10x(2x^2 - 3)$
$f''(-2) = 10(-2)(8 - 3) = 10(-)(+)$
$f''(-1) = 10(-1)(2 - 3) = 10(-)(-)$
$f''(1) = 10(1)(2 - 3) = 10(+)(-)$
$f''(2) = 10(2)(8 - 3) = 10(+)(+)$

2. Let $g(x) = \ln(x^2 + 4)$. Note $g'(x) = \dfrac{2x}{x^2 + 4}$ and $g''(x) = -\dfrac{2\left(x^2 - 4\right)}{\left(x^2 + 4\right)^2}$

   (a) Determine the domain of $g(x)$. Domain is $x^2 + 4 > 0 \Rightarrow x^2 > -4$ which is always!
   (b) Find the intervals of increase or decrease.
   (c) Find the local maximum and minimum values.
   (d) Find the intervals of concavity and inflection points.
   (e) Use the information to sketch the graph.

Critical points: $g'(x) = 0 \Rightarrow 2x = 0 \Rightarrow \boxed{x = 0}$
$g'(x)$ DNE $\Rightarrow x^2 + 4 = 0 \Rightarrow x^2 = -4$ None!

Possible infl pts: $g''(x) = 0 \Rightarrow x^2 - 4 = 0 \Rightarrow \boxed{x = 2}$ or $\boxed{x = -2}$
$g''(x)$ DNE $\Rightarrow (x^2 + 4)^2 = 0 \Rightarrow x^2 = -4$ None.



$g(0) = \ln(0^2 + 4) = \ln(4)$

Table

x		-2		0		2	
Sample	-3		-1		1		3
Sign g'	−		−	0	+		+
Incr/decr	↘		↘	MIN	↗		↗
Sign g''	−	0	+	+	+	0	−
Concavity	∩	INF	∪	∪	∪	INF	∩

$g'(-1) = \dfrac{2(-1)}{(-1)^2 + 4} = \dfrac{-}{+} = -$

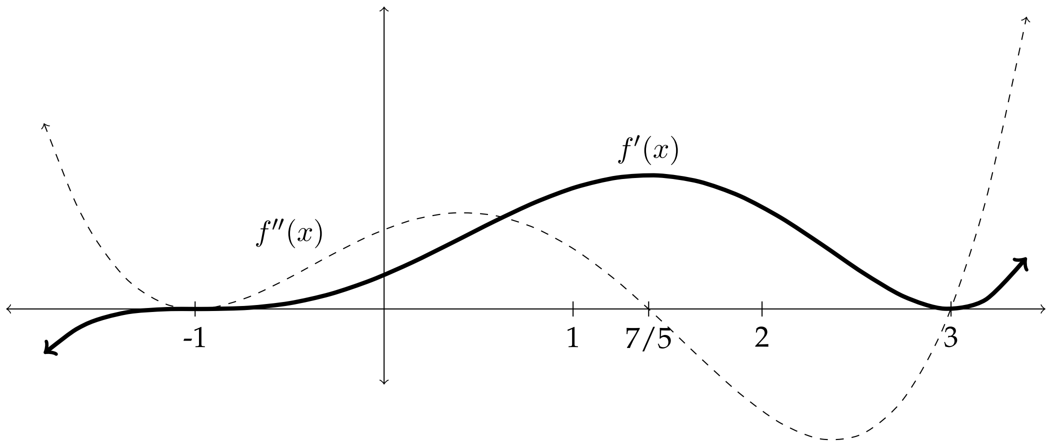
$g'(1) = \dfrac{2(1)}{1^2 + 4} = \dfrac{+}{+} = +$

$g''(-3) = \dfrac{-2((-3)^2 - 4)}{+} = \dfrac{(-2)(+)}{+} = -$

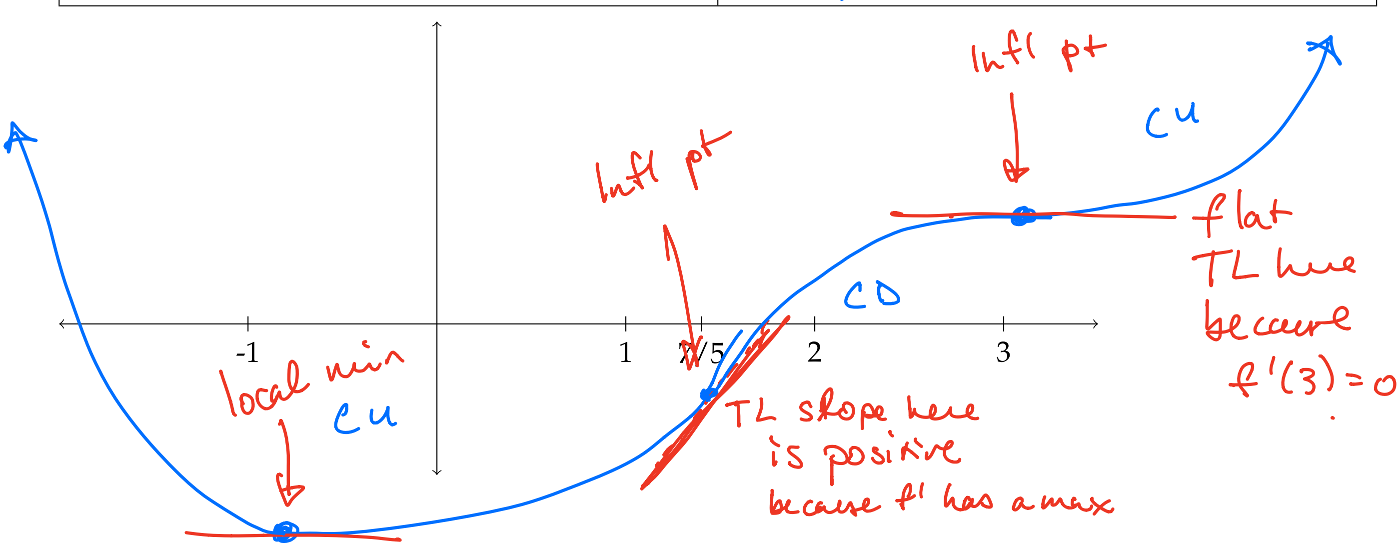
$g''(0) = \dfrac{-2(0^2 - 4)}{+} = \dfrac{(-)(-)}{+} = +$

$g''(3) = \dfrac{-2(+)}{+} = -$

3. Below are the graphs of the FIRST DERIVATIVE, $f'(x)$, and the SECOND DERIVATIVE, $f''(x)$, of some unknown function f. Note that $f'(x)$ is the solid curve and $f''(x)$ is the dashed curve. (Assume the domain of all the functions is $(-\infty, \infty)$ and that the functions continue in the way that they are going outside the area shown.) Identify all the information about $f(x)$ required in the table and then sketch the graph of $f(x)$ on the given axes.



property of f	**interval or point(s)**
f is increasing $f' > 0$	$(-1, \infty)$
f is decreasing $f' < 0$	$(-\infty, -1)$
f has a local maximum	None
f has a local minimum ↘ ↗	-1
f is concave up $f'' > 0$	$(-\infty, 7/5) \cup (3, \infty)$
f is concave down $f'' < 0$	$(7/5, 3)$
f has an inflection point	$7/5$, 3



We don't know where $f(0)$ is. That's ok.