SECTION 4.8 NEWTON'S METHOD

Newton's Method is an iterative rule for finding roots.

Given: $F(x)$
Want: a so that $F(a) = 0$
Guess: x_0 close to a

Plug in and Repeat:
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newX = oldX - F(oldX)/F'(oldX)
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In math language:

$$x_{k+1} = x_k - \frac{F(x_k)}{F'(x_k)}$$

1. Let $F(x) = x^2 - 2$.

   (a) Using elementary algebra, find a such that $F(a) = 0$. (Find a exactly and find a decimal approximation with at least 9 decimal places.)

   (b) Find a formula for x_{k+1}. Simplify it.

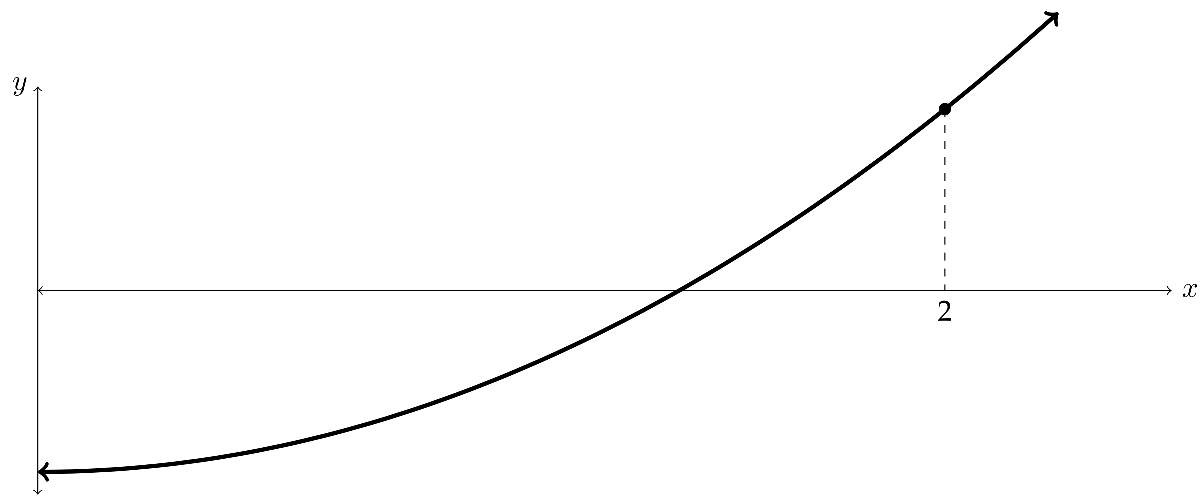
   (c) Using an initial guess of $x_0 = 2$, complete 4 iterations of Newton's method to find x_4 and compare your answer to the one in part (a).

2. This page is intended to illustrate *how* Newton's Method works.

   Again, consider the function $F(x) = x^2 - 2$.

   (a) Find the linearization $L(x)$ of $F(x)$ at $x = 2$. Leave your answer in point-slope form.

   (b) I've graphed $F(x)$ for you below. Mark where $\sqrt{2}$ is on this diagram and add to this diagram the graph of $L(x)$. Use a ruler.



   (c) Find the number x_1 such that $L(x_1) = 0$.

   (d) In the diagram above, label the point x_1 on the x-axis.

   (e) Let's do it again! Find the linearization $L(x)$ of $F(x)$ at $x = x_1$.

   (f) Add the graph of this new linearization to your diagram above.

   (g) Find the number x_2 such that $L(x_2) = 0$. Then label the point $x = x_2$ in the diagram.

   (h) Compare your numbers for x_1 and x_2 to those on the previous page. They should be the same.

(i) Let's be a little more systematic. Suppose we have an estimate x_k for $\sqrt{2}$.

- Compute $F(x_k)$.
- Compute $F'(x_k)$.
- Compute the linearization of $F(x)$ at $x = x_k$.

  $L(x) =$

- Find the number x_{k+1} such that $L(x_{k+1}) = 0$. You should try to find as simple an expression as you can. Compare this to the formula we used on problem 1b from page 1.

3. Try to solve

$$e^{-x} - x = 0$$

by hand.

4. Explain why there is a solution between $x = 0$ and $x = 1$.

5. Starting with $x_0 = 1$, find an approximation of the solution of $e^{-x} - x = 0$ to 6 decimal places. During your computation, keep track of each x_k to at least 9 decimal places of accuracy.