SECTION 4.9: ANTIDERIVATIVES

1. Find a particular antiderivative of $f(x) = 9 + x - x^2$.

$F(x) = 9x + \frac{x^2}{2} - \frac{x^3}{3} + 12$

check: $F'(x) = 9 + \frac{2x}{2} - \frac{3x^2}{3} + 0 = 9 + x - x^2$ ✓

2. Find all antiderivatives of $f(x) = 9 + x - x^2$.

$F(x) = 9x + \frac{x^2}{2} - \frac{x^3}{3} + C$ where C is a constant

3. Find an antiderivative of $f(x) = \frac{1}{x^2}$. $= x^{-2}$

$F(x) = -x^{-1} = -\frac{1}{x}$

$F'(x) = -1(-x^{-2}) = \frac{1}{x^2}$ ✓

4. To find *all* antiderivatives of a function $f(x)$, do you always just add a $+C$? Explain how to construct a "generic" piecewise function where you're not using just $+C$ to describe *all* antiderivatives.

$f(x) = \begin{cases} 2x & x \geq 0 \\ -2x & x < 0 \end{cases}$ $F(x) = \begin{cases} x^2 + C \\ -x^2 + D \end{cases}$ C, D not the same constant!

Check: $F'(x) = \begin{cases} 2x & x \geq 0 \\ -2x & x < 0 \end{cases}$ ✓

5. For each of the following functions, find a particular antiderivative.

Function	Antiderivative
x	$x^2/2$
x^2	$x^3/3$
x^3	$x^4/4$
x^k $(k \neq -1)$	$x^{k+1}/(k+1)$
x^{-1} for $x > 0$	$\ln(x)$
x^{-1} for $x < 0$	$\ln(-x)$
x^{-1} for all x	$\ln\lvert x\rvert$

Function	Antiderivative
$\sin(x)$	$-\cos(x)$
$\cos(x)$	$\sin(x)$
e^x	e^x
$1/(1+x^2)$	$\arctan(x)$
$(\sec(x))^2$	$\tan(x)$
$\sec(x)\tan(x)$	$\sec(x)$
1	x

6. Compute an antiderivative of $f(x) = 15x^{20} + 44x^{10} + 8$

$F(x) = \frac{15 \cdot x^{21}}{21} + \frac{44 \cdot x^{11}}{11} + 8x = \frac{5}{7}x^{21} + 4x^{11} + 8x$

Check: $F'(x) = \frac{5}{7}(21)x^{20} + 4(11)x^{10} + 8 = 15x^{20} + 44x^{10} + 8$ ✓

7. Compute an antiderivative of $f(t) = \frac{5\sec t \tan t}{3} - 4\sin t - \frac{1}{t} + e^2$

$F(t) = \frac{5}{3}\sec(t) + 4\cos(t) - \ln|t| + e^2 t$

Check: $F'(t) = \frac{5}{3}\sec(t)\tan(t) + 4(-\sin(t)) - \frac{1}{t} + e^2$ ✓

8. Compute an antiderivative of $f(x) = \cos(3x)$.

$F(x) = \frac{\sin(3x)}{3}$

Check: $F'(x) = \frac{1}{3}(\cos(3x))(3) = \cos(3x)$ ✓

9. Compute the antiderivative of $f(t) = t^2$ that equals 5 when $t = 2$.

$F(t) = \frac{t^3}{3} + C$

Need $F(2) = 5 \Rightarrow 5 = \frac{8}{3} + C \Rightarrow \frac{15}{3} - \frac{8}{3} = C$

$\Rightarrow C = \frac{7}{3}$.

So $\boxed{F(t) = \frac{t^3}{3} + \frac{7}{3}}$