SECTION 5-3: THE FUNDAMENTAL THEOREM OF CALCULUS, PART 1

1. Suppose f is the function whose graph is shown and that $g(x) = \int_0^x f(t)dt$.

   (a) Find the values of $g(0)$, $g(1)$, $g(2)$, $g(3)$, $g(4)$, $g(5)$, and $g(6)$. Then, sketch a rough graph of g.

   (a) $g(0) =$ 0

   (b) $g(1) =$ 1

   (c) $g(2) =$ 3

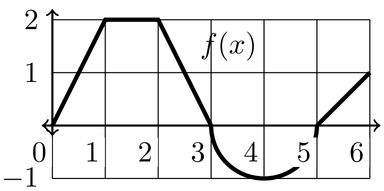
   (d) $g(3) =$ 4

   (e) $g(4) =$ $4 - \pi/4 \approx 3.21$

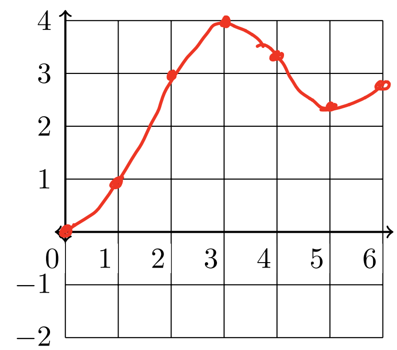
   (f) $g(5) =$ $4 - \pi/2 \approx 2.42$

   (g) $g(6) =$ $4 - \pi/2 + 1/2 \approx 2.92$

Area of circle w/radius 1 is π



Sketch of $g(x)$



   (i) Where is $g(x)$ increasing? $(0,3) \cup (5,6)$

   (ii) Describe f when $g(x)$ is increasing. positive

   (iii) Where is $g(x)$ decreasing? $(3,5)$

   (iv) Describe f when $g(x)$ is decreasing. negative

   (v) Where does $g(x)$ have a local maximum? $x = 3$

   (vi) Describe f when $g(x)$ has a local max. $f(x) = 0$ and f goes from + to −

   (vii) Where does $g(x)$ have a local minimum? $x = 5$

   (viii) Describe f when $g(x)$ has a local min. $f(x) = 0$ and f goes from − to +

   (b) Make a guess: what is the relationship between $g(x)$ and $f(x)$?

   $f(x) = g'(x)$

The Fundamental Theorem of Calculus, Part 1 If f is continuous on $[a, b]$, the function g defined by

$$g(x) = \int_a^x f(t)\, dt \quad a \leq x \leq b$$

is continuous on $[a, b]$ and differentiable on (a, b) and $g'(x) = f(x)$.

2. Find the derivative of $g(x) = \int_2^x t^2\, dt$.

$g'(x) = x^2$ ← careful of your variable name!

3. The Fresnel function $S(x) = \int_0^x \sin(\pi t^2/2)\, dt$ first appeared in Fresnel's theory of the diffraction of light waves. Recently it was be applied to the design of highways. Find the derivative of the Fresnel function.

$S'(x) = \sin\left(\frac{\pi x^2}{2}\right)$

4. Consider $g(x) = \int_1^{x^4} \sec t\, dt$.

   Let $u = x^4$ and $h(x) = \int_1^x \sec t\, dt$.

   (a) Write $g(x)$ as a composition.

   $u = x^4$, so

   $g(x) = h(u(x))$.

   Note $h'(u) = \sec(u)$

   by FTC1.

   (b) Use FTC1 and the chain rule to differentiate $g(x)$.

   So

   $g'(x) = h'(u) \cdot \frac{du}{dx}$

   $= \sec(x^4) \cdot 4x^3$

5. Consider $g(x) = \int_{2x+1}^{2} \sqrt{t}\, dt$.

   (a) Write $g(x)$ as a composition.

   $u = 2x+1$, $h(u) = -\int_2^u \sqrt{t}\, dt$

   So $g(x) = h(u(x))$

   Notice we flipped the integral when defining h because FTC1 says if $g(x) = \int_a^x f(t)\,dt$ then $g'(x) = f(x)$.

   (b) Use FTC1 and the chain rule to differentiate $g(x)$.

   So $g(x) = h(u(x))$ and

   $g'(x) = h'(u) \cdot \frac{du}{dx}$

   $= (\sqrt{2x+1})(2)$

6. Consider the function $g(x) = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}}\, dt$. Observe that

$$\int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}}\, dt = \int_{\tan x}^{0} \frac{1}{\sqrt{2+t^4}}\, dt + \int_0^{x^2} \frac{1}{\sqrt{2+t^4}}\, dt.$$

Use properties of definite integrals, FTC1, and the chain rule to determine $g'(x)$. Let $u = \tan(x)$, $v = x^2$

$g(x) = -\int_0^{\tan x} \frac{1}{\sqrt{2+t^4}}\, dt + \int_0^{x^2} \frac{1}{\sqrt{2+t^4}}\, dt = -\int_0^u \frac{1}{\sqrt{2+t^4}}\, dt + \int_0^v \frac{1}{\sqrt{2+t^4}}\, dt \Rightarrow$

$g'(x) = -\frac{1}{\sqrt{2+u^4}} \frac{du}{dx} + \frac{1}{\sqrt{2+v^4}} \frac{dv}{dx} = \frac{-(\sec(x))^2}{\sqrt{1+(\tan(x))^4}} + \frac{2x}{\sqrt{2+(x^2)^4}}$