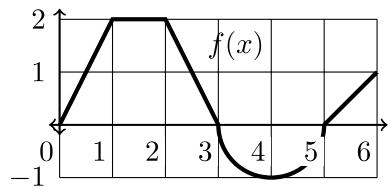
SECTION 5-3: THE FUNDAMENTAL THEOREM OF CALCULUS, PART 1

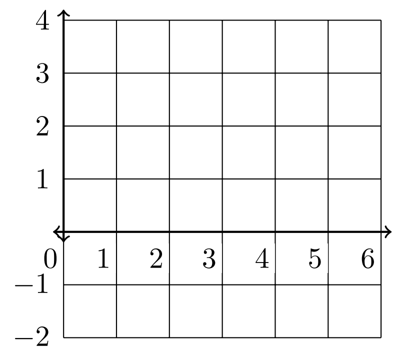
1. Suppose f is the function whose graph is shown and that $g(x) = \int_0^x f(t)dt$.

   (a) Find the values of $g(0)$, $g(1)$, $g(2)$, $g(3)$, $g(4)$, $g(5)$, and $g(6)$. Then, sketch a rough graph of g.

      (a) $g(0) =$ ________________
      (b) $g(1) =$ ________________
      (c) $g(2) =$ ________________
      (d) $g(3) =$ ________________
      (e) $g(4) =$ ________________
      (f) $g(5) =$ ________________
      (g) $g(6) =$ ________________



Sketch of $g(x)$



      (i) Where is $g(x)$ increasing? ________________
      (ii) Describe f when $g(x)$ is increasing. ________________
      (iii) Where is $g(x)$ decreasing? ________________
      (iv) Describe f when $g(x)$ is decreasing. ________________
      (v) Where does $g(x)$ have a local maximum? ________________
      (vi) Describe f when $g(x)$ has a local max. ________________
      (vii) Where does $g(x)$ have a local minimum? ________________
      (viii) Describe f when $g(x)$ has a local min. ________________

   (b) Make a guess: what is the relationship between $g(x)$ and $f(x)$?

The Fundamental Theorem of Calculus, Part 1 If f is continuous on $[a, b]$, the function g defined by

$$g(x) = \int_a^x f(t)\,dt \quad a \leq x \leq b$$

is continuous on $[a, b]$ and differentiable on (a, b) and $g'(x) = f(x)$.

2. Find the derivative of $g(x) = \int_2^x t^2\,dt$.

3. The Fresnel function $S(x) = \int_0^x \sin(\pi t^2/2)\,dt$ first appeared in Fresnel's theory of the diffraction of light waves. Recently it was be applied to the design of highways. Find the derivative of the Fresnel function.

4. Consider $g(x) = \int_1^{x^4} \sec t\,dt$.

   Let $u = x^4$ and $h(x) = \int_1^x \sec t\,dt$.

   (a) Write $g(x)$ as a composition.

   (b) Use FTC1 and the chain rule to differentiate $g(x)$.

5. Consider $g(x) = \int_{2x+1}^{2} \sqrt{t}\,dt$.

   (a) Write $g(x)$ as a composition.

   (b) Use FTC1 and the chain rule to differentiate $g(x)$.

6. Consider the function $g(x) = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}}\,dt$. Observe that

$$\int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}}\,dt = \int_{\tan x}^{0} \frac{1}{\sqrt{2+t^4}}\,dt + \int_{0}^{x^2} \frac{1}{\sqrt{2+t^4}}\,dt.$$

   Use properties of definite integrals, FTC1, and the chain rule to determine $g'(x)$.