SECTION 5-3: THE FUNDAMENTAL THEOREM OF CALCULUS, PART 2

The Fundamental Theorem of Calculus (Part 2) If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where F is **any antiderivative** of f, that is, is a function such that $F' = f$. To evaluate, we write

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a).$$

1. Evaluate the following integrals.

   (a) $\displaystyle\int_0^1 x^2\,dx$

   (b) $\displaystyle\int_1^4 (1 + 3y - y^2)\,dy$

2. Review from §4.9: To compute integrals effectively you **must** have your basic antidifferentiation formulas down. You should know that antiderivatives to the following functions. We are using the $\int$ symbol to mean "find the antiderivative" of the function right after the symbol.

Antiderivatives of common functions:

- $\displaystyle\int x^n\,dx =$ _____
- $\displaystyle\int \sin x\,dx =$ _____
- $\displaystyle\int \cos x\,dx =$ _____
- $\displaystyle\int \sec^2 x\,dx =$ _____
- $\displaystyle\int \sec x \tan x\,dx =$ _____
- $\displaystyle\int \csc^2 x\,dx =$ _____
- $\displaystyle\int \csc x \cot x\,dx =$ _____
- $\displaystyle\int e^x\,dx =$ _____
- $\displaystyle\int a^x\,dx =$ _____
- $\displaystyle\int \frac{1}{1+x^2}\,dx =$ _____
- $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx =$ _____
- $\displaystyle\int \frac{1}{x}\,dx =$ _____

3. Evaluate the following integrals.

   (a) $\displaystyle\int_2^5 \frac{3}{x}\,dx$

   (b) $\displaystyle\int_0^{\pi/2} \cos x\,dx$

4. Evaluate the following integrals.

(a) $\displaystyle\int_1^8 \sqrt[3]{x}\,dx$ (b) $\displaystyle\int_{\pi/6}^{\pi/2} \csc x \cot x\,dx$ (c) $\displaystyle\int_0^1 \frac{9}{1+x^2}\,dx$

5. We do not have any product or quotient rules for antidifferentiation. To evaluate an integral that is expressed as a product or quotient you must try to manipulate the integrand (the stuff inside the $\int$ sign) to look like something you know how to antidifferentiate. The following integrals are examples of this. Evaluate the following integrals.

(a) $\displaystyle\int_1^3 \frac{x^3+3x^6}{x^4}\,dx$ (b) $\displaystyle\int_0^1 x(3+\sqrt{x})\,dx$

6. Evaluate the following integrals.

(a) $\displaystyle\int_0^2 (5^x + x^5)\,dx$ (b) $\displaystyle\int_{1/2}^{\sqrt{2}/2} \frac{1}{\sqrt{1-x^2}}\,dx$

7. What is wrong with the following calculation? (Hint: draw a picture!)

$$\int_{-1}^{3} \frac{1}{x^2}\,dx = \left.\frac{x^{-1}}{-1}\right|_{-1}^{3} = -\frac{1}{3} - 1 = -\frac{4}{3}$$