SECTION 5-4: INDEFINITE INTEGRALS AND THE NET CHANGE THEOREM

1. Compute $\int x^2(3-x)\,dx$

$$= \int 3x^2 - x^3\,dx = \frac{3x^3}{3} - \frac{x^4}{4} + C$$

2. Compute $\int 9\sqrt{x} - 3\sec(x)\tan(x)\,dx$

$$= 9x^{3/2}\cdot\frac{2}{3} - 3(\sec(x))^2 + C$$

3. Snow is falling on my garden at a rate of

$$A(t) = 10e^{-2t}$$

kilograms per hour for $0 \le t \le 2$, where t is measured in hours.

(a) Find $A(1)$ and interpret in the context of the problem.

$A(1) = 10e^{-2(1)} = \frac{10}{e^2} \approx 1.35$ At one hour, the snow is falling at a rate of $\frac{10}{e^2} \approx 1.35$ kilograms/hour

(b) If $m(t)$ is the total mass of snow on my garden, how are $m(t)$ and $A(t)$ related to each other?

$m(t) =$ initial mass of snow $+ \int_0^t A(x)\,dx$

$A(t)$ is the derivative of $m(t)$ at least for $0 \le t \le 2$

(c) What does $m(2) - m(0)$ represent?

The total snow accumulation over the 2 hours

(d) Find an antiderivative of $A(t)$.

$$\int A(t)\,dt = \frac{10e^{-2t}}{-2} + C$$

Check: $\frac{d}{dt}\left(\frac{10e^{-2t}}{-2}\right) = -\frac{10}{2}e^{-2t}(-2) = -10e^{-2t}$

(e) Compute the total amount of snow accumulation from $t = 0$ to $t = 1$.

$$\int_0^1 A(t)\,dt = -5e^{-2t}\Big|_0^1 = -5e^{-2} - (-5e^0) = \frac{-5}{e^2} + 5 \approx 4.32 \text{ kg}$$

(f) Compute the total amount of snow accumulation from $t = 0$ to $t = 2$.

$$\int_0^2 A(t)\,dt = -5e^{-2t}\Big|_0^2 = 5 - \frac{5}{e^4} \approx 4.90 \text{ kg}$$

(g) From the information given so far, can you compute $m(2)$?

No! I have no idea how much snow mass was already on my garden.

(h) Suppose $m(0) = 9$. Compute $m(1)$ and $m(2)$.

$m(1) = 9 + 5 - \frac{5}{e^2} = 14 - \frac{5}{e^2}$

$m(2) = 9 + 5 - \frac{5}{e^4} =$

4. A airplane is descending. Its **rate of change** of height is $r(t) = -4t + \frac{t^2}{10}$ meters per second.

(a) if $A(t)$ is the altitude of the airplane in meters, how are $A(t)$ and $r(t)$ related?

$r(t)$ is the derivative of $A(t)$

(b) What physical quantity does $\int_1^3 r(t)\,dt$ represent?

The change in height from $t=1$ to $t=3$

(that is, $\int_1^3 r(t)\,dt = A(3) - A(1)$

(c) Compute $A(3) - A(1)$.

$\int r(t)\,dt = -\frac{4t^2}{2} + \frac{t^3}{3 \cdot 10} = -2t^2 + \frac{t^3}{30}$

$A(3) - A(1) = \left(-2(9) + \frac{27}{30}\right) - \left(-2 + \frac{1}{30}\right) = -16 + \frac{26}{30} = -16 + \frac{13}{15} \approx -15.13$

(d) Can we determine the height of the plane when $t = 3$? If so, determine it; if not, explain why.

No, we don't know how high it was at $t=1$. All we know is that its height decreased by a bit over 15 meters over the two seconds.

5. Gravel is being added to a pile at a rate of rate of $1 + t^2$ tons per minute for $0 \le t \le 10$ minutes. If $G(t)$ is the amount of gravel (in tons) in the pile at time t, compute $G(10) - G(0)$.

$G(10) - G(0) = \int_0^{10} 1 + t^2\,dt = t + \frac{t^3}{3}\Big|_0^{10} = \left(10 + \frac{1000}{3}\right) - 0$

$= \frac{1030}{3}$

Over the 10 minutes, $343\frac{1}{3}$ tons of gravel was added to the pile.