SECTION 5-5: SUBSTITUTION (DAY 2)

1. Compute $\int \frac{\sec^2(x)}{\tan(x)}\,dx$

Let $u = \tan x$. Then $\frac{du}{dx} = (\sec(x))^2 \Rightarrow \frac{du}{(\sec(x))^2} = dx$.

So $\int \frac{(\sec(x))^2}{\tan(x)}\,dx = \int \frac{(\sec(x))^2}{u} \cdot \frac{du}{(\sec(x))^2} = \int \frac{du}{u} = \ln|u| + c$

$= \boxed{\ln|\tan(x)| + c.}$

2. Compute $\int \sec^2(x)\tan(x)\,dx = \int \sec(x)(\sec(x)\tan(x))\,dx$.

Let $u = \sec(x)$. Then $du = \sec(x)\tan(x)\,dx$ so

$\int \sec(x) \cdot \sec(x)\tan(x)\,dx = \int u\,du = \frac{u^2}{2} + c = \boxed{\frac{(\sec(x))^2}{2} + c.}$

3. Compute $\int \frac{\sin(\theta)}{1+\cos(\theta)}\,d\theta$

Let $u = 1 + \cos\theta$. Then $\frac{du}{d\theta} = -\sin\theta \Rightarrow \frac{du}{-\sin\theta} = d\theta$. So

$\int \frac{\sin\theta}{1+\cos\theta}\,d\theta = \int \frac{\sin\theta}{u} \cdot \frac{du}{-\sin\theta} = -\int \frac{1}{u}\,du$

$= -\ln|u| + c = \boxed{-\ln|1+\cos\theta| + c.}$

4. Compute $\int \frac{1}{x\ln(x)}\,dx$

Let $u = \ln(x)$. Then $\frac{du}{dx} = \frac{1}{x}$ so $x\,du = dx$. Thus

$$\int \frac{1}{x\ln(x)}\,dx = \int \frac{1}{\cancel{x}\cdot u}(\cancel{x}\,du) = \int \frac{1}{u}\,du = \ln|u| + c = \boxed{\ln|\ln(x)| + c.}$$

5. Compute $\int \frac{\sin(4/x)}{x^2}\,dx$

Let $u = 4/x = 4x^{-1}$. Then $\frac{du}{dx} = -4x^{-2} = \frac{-4}{x^2}$. So $\frac{x^2\,du}{-4} = dx$.

Therefore $\int \frac{\sin(4/x)}{x^2}\,dx = \int \frac{\sin(u)}{\cancel{x^2}}\left(\frac{\cancel{x^2}\,du}{-4}\right) = -\frac{1}{4}\int \sin(u)\,du$

$$= -\frac{1}{4}(-\cos(u)) + c = \boxed{\frac{1}{4}\cos(4/x) + c}$$

Check: $\frac{d}{dx}\left(\frac{1}{4}\cos(4/x) + c\right) = -\frac{1}{4}\sin(4/x)\left(\frac{-4}{x^2}\right) = \frac{\sin(4/x)}{x^2}$ ✓

6. Compute $\int \frac{e^x}{e^x - 3}\,dx$

Let $u = e^x - 3$. Then $du = e^x\,dx$. So

$$\int \frac{e^x\,dx}{e^x - 3} = \int \frac{1}{u}\,du = \ln|u| + c = \boxed{\ln|e^x - 3| + c}$$

7. Compute $\int \frac{1}{9+x^2}\,dx$ $= \int \frac{1}{9\left(1+\frac{x^2}{9}\right)}\,dx = \int \frac{1}{9\left(1+\left(\frac{x}{3}\right)^2\right)}\,dx$

$= \frac{1}{9}\int \frac{1}{1+\left(\frac{x}{3}\right)^2}\,dx.$ Let $u = \frac{x}{3} \Rightarrow \frac{du}{dx} = \frac{1}{3} \Rightarrow 3\,du = dx.$

So $\frac{1}{9}\int \frac{1}{1+\left(\frac{x}{3}\right)^2}\,dx = \frac{1}{9}\int \frac{1}{1+u^2}(3\,du) = \frac{1}{3}\int \frac{1}{1+u^2}\,du$

$= \frac{1}{3}\arctan(u) + C = \boxed{\frac{1}{3}\arctan\left(\frac{x}{3}\right) + C.}$

8. Compute $\int \sqrt{x}(x^4+x)\,dx$

$= \int x^{1/2}(x^4+x)\,dx = \int x^{17/2} + x^{3/2}\,dx$

$= \frac{x^{19/2}}{19/2} + \frac{x^{5/2}}{5/2} + C = \boxed{\frac{2x^{19/2}}{19} + \frac{2x^{5/2}}{5} + C.}$

9. Compute $\int \cos(x)\sin(\sin(x))\,dx$

Let $u = \sin(x)$. Then $\frac{du}{dx} = \cos(x) \Rightarrow \frac{du}{\cos(x)} = dx.$ So

$\int \cos(x)\sin[\sin(x)]\,dx = \int \cancel{\cos(x)}\sin(u)\left(\frac{du}{\cancel{\cos(x)}}\right) = \int \sin(u)\,du$

$= -\cos(u) + C = \boxed{-\cos(\sin(x)) + C}$

10. Compute $\frac{d}{dx}\left[x\ln(x)-x\right]$. Then compute $\int s^2\ln(s^3)\,ds$

$\frac{d}{dx}\left(x\ln(x)-x\right) = x\cdot\frac{1}{x} + \ln(x) - 1 = \ln(x).$

To compute $\int s^2\ln(s^3)\,ds$, let $u=s^3$. Then $\frac{du}{ds}=3s^2 \Rightarrow \frac{du}{3s^2}=ds.$

So $\int s^2\ln(s^3) = \int s^2\ln(u)\cdot\frac{du}{3s^2} = \frac{1}{3}\int\ln(u)\,du.$

Observe $\frac{d}{dx}(x\ln(x)-x)=\ln(x)$, so $\int\ln(x)\,dx = x\ln(x)-x+C$. So

$\frac{1}{3}\int\ln(u)\,du = \frac{1}{3}(u\ln(u)-u)+C = \boxed{\frac{1}{3}\left(s^3\ln(s^3)-s^3\right)+C.}$

11. Compute $\int x\sqrt{x-1}\,dx$ (Hint: Let $u=x-1$. What is x in terms of u?)

Let $u=x-1$. Then $x=u+1$, and $dx=du$. So

$\int x\sqrt{x-1}\,dx = \int(u+1)\sqrt{u}\,du = \int u^{3/2}+u^{1/2}\,du$

$= \frac{2}{5}u^{5/2}+\frac{2}{3}u^{3/2}+C$

$= \boxed{\frac{2}{5}(x-1)^{5/2}+\frac{2}{3}(x-1)^{3/2}+C}$

12. Compute $\int_1^3 \frac{(\ln(x))^3}{x}\,dx$. Let $u=\ln(x)$. Then $\frac{du}{dx}=\frac{1}{x} \Rightarrow x\,du=dx.$

If $x=1$, $u=\ln(1)$ & if $x=3$, $u=\ln(3)$. So

$\int_1^3\frac{(\ln(x))^3}{x}\,dx = \int_{\ln(1)}^{\ln(3)}\frac{u^3}{x}(x\,du) = \int_{\ln(1)}^{\ln(3)}u^3\,du = \left.\frac{u^4}{4}\right|_{\ln(1)}^{\ln(3)}$

$= \frac{(\ln(3))^4}{4} - 0 = \boxed{\frac{(\ln(3))^4}{4}}.$