# §2-3: Common Limit Mistakes

Each of the following limits has some common mistake. Identify the mistakes!

Hint: Most of the mistakes are mistakes in notation. In particular: are you *saying* you're taking the limit after you've evaluated it? Alternately, are you still working on a limit but there's no limit notation? When things are equal, are using an $=$ to say so?

1.

$$\lim_{x\to -2}(2x^2-10x+1) = \lim_{x\to -2}(2(-2)^2-10(-2)+1) = \lim_{x\to -2}(8+20+1) = 29$$

2.

$$\lim_{x\to 1}\frac{4-4x^2}{5x-5} = \frac{4(1-x^2)}{5(x-1)} = \frac{-4(x^2-1)}{5(x-1)} = \frac{-4\cancel{(x-1)}(x+1)}{5\cancel{(x-1)}} = \frac{-4(x+1)}{5} = \frac{-8}{5}$$

3.

$$\begin{aligned}\lim_{x\to 2}\frac{2x^2-8x+8}{x^2-4} &= \lim_{x\to 2}\frac{2(x^2-4x+4)}{x^2-4}\\ &= \lim_{x\to 2}\frac{2(x-2)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}\\ &= \lim_{x\to 2}\frac{2(x-2)}{x+2}\\ &= \lim_{x\to 2}0\\ &= 0\end{aligned}$$

4.

$$\lim_{x\to 1}\frac{4-4x^2}{5x-5} = \lim_{x\to 1}\frac{4(1-x^2)}{5(x-1)} = \lim_{x\to 1}\frac{4\cancel{(1-x)}(1+x)}{5\cancel{(x-1)}} = \lim_{x\to 1}\frac{4(x+1)}{5} = \frac{8}{5}$$

5.

$$\begin{aligned}\lim_{x\to 2}\frac{2-\sqrt{x+2}}{x-2} &= \left(\frac{2-\sqrt{x+2}}{x-2}\right)\left(\frac{2+\sqrt{x+2}}{2+\sqrt{x+2}}\right)\\ &= \frac{4-x-2}{(x-2)(2+\sqrt{x+2})}\\ &= \frac{2-x}{(x-2)(2+\sqrt{x+2})}\\ &= \frac{-1}{2+\sqrt{x+2}}\\ &= \frac{-1}{4}\end{aligned}$$

6.

$$\begin{aligned}&\lim_{x\to 2}\sqrt{2x^2-2x+2}\\ &\sqrt{2(2)^2-2(2)+2}\\ &\sqrt{8-4+2}\\ &\sqrt{6}\end{aligned}$$

7. $\lim_{x\to -2}(2^x - 5x + 1) \implies (2^3 - 5(3) + 1) \implies 8 = 15 + 1 \implies -6$

8.

$$\begin{aligned}\lim_{x\to 2}\frac{2-\sqrt{x+2}}{x-2} &= \lim_{x\to 2}\left(\frac{2-\sqrt{x+2}}{x-2}\right)\left(\frac{2+\sqrt{x+2}}{2+\sqrt{x+2}}\right)\\ &= \lim_{x\to 2}\frac{4-\cancel{x-2}}{\cancel{(x-2)}(2+\sqrt{x+2})}\\ &= \lim_{x\to 2}\frac{4}{2+\sqrt{x+2}}\\ &= 1\end{aligned}$$

9. $\lim_{x\to 4}\dfrac{2x-8}{16-x^2} \qquad \lim_{x\to 4}\dfrac{2(x-4)}{-(x-4)(x+4)} \qquad \lim_{x\to 4}\dfrac{2}{-8} = \dfrac{-1}{4}$

10.

$$\begin{aligned}\lim_{x\to 2}\frac{2x^2-x-1}{4x^2-1} = \frac{2(2)^2-2-1}{4(2)^2-1} = \frac{2(4)-3}{4(2^2)-1} &= \frac{5}{15}\\ \lim_{x\to 2} &= \frac{1}{3}\end{aligned}$$

1. You continued to use $\lim_{x\to ??}$(blah) after you'd evaluated the limit.
2. You're still taking limits! Those things that you say are equal are not really equal. Need more limits.
3. Almost perfect—except for that extraneous last $\lim_{x\to 2} 0$.
4. $(1-x)$ and $(x-1)$ aren't equal, so you can't cancel them. You need to pull a $-1$ out of one of them first. ***Incorrect algebra; wrong limit.***
5. You need to have the limits until you evaluate them. This is super-common and super-WRONG.
6. You need to say things are equal by using $=$. This really bugs me.
7. If things are equal, indicate so by using $=$, not $\implies$ . $\implies$ means something else (e.g., something logically follows from something).
8. Algebra-foul. You just can't do that. ***Incorrect algebra; wrong limit.***
9. If things are equal, say so with $=$, not just hoping I get it because the terms are in vague proximity. Also, there's an extra last limit.
10. $\lim_{x\to 2}(\dots)$ is a *function* which eats other functions and spits out numbers, $\pm\infty$, or *DNE*. It has no meaning by itself.