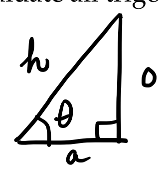
LECTURE: 1-5: TRIGONOMETRY REVIEW

Basic Trigonometry

You want to recall:
(a) the triangle definitions of all six trigonometric functions
(b) the definitions of the four non-sine and cosine trigonometric functions in terms of sine and cosine
(c) be able to graph all six trigonometric functions
(d) be familiar with the unit circle definition and be able to evaluate all trigonometric functions at common angles without the use of a calculator
(e) remember the Pythagorean Identities.



The Triangle Defintion

Example 1: Sketch a right triangle with side a adjacent to an angle θ, o opposite of the angle θ and hypotenuse h. Define each of the six trigonometric functions in terms of that triangle.

a) $\sin\theta$ $\quad \frac{o}{h}$

b) $\cos\theta$ $\quad \frac{a}{h}$

c) $\tan\theta$ $\quad \frac{o}{a}$

d) $\sec\theta$ $\quad \frac{h}{a}$

e) $\csc\theta$ $\quad \frac{h}{o}$

f) $\cot\theta$ $\quad \frac{a}{o}$

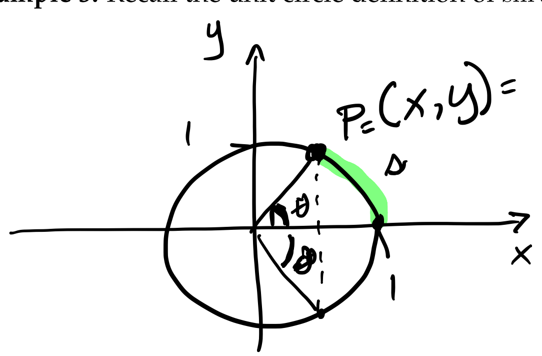
Functions in Terms of Sine and Cosine

Example 2: Define the following four functions in terms of sine and cosine.

(a) $\tan\theta = \frac{\sin\theta}{\cos\theta}$ (b) $\sec\theta = \frac{1}{\cos\theta}$ (c) $\csc\theta = \frac{1}{\sin\theta}$ (d) $\cot\theta = \frac{\cos\theta}{\sin\theta}$

The Unit Circle Approach

Example 3: Recall the unit circle definition of $\sin\theta$ and $\cos\theta$.

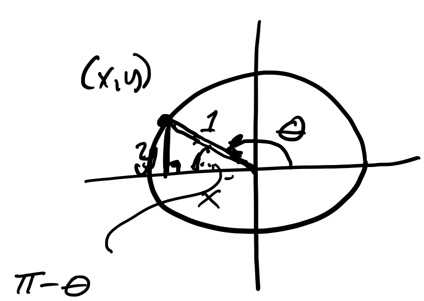


$P = (x, y) = (\cos\theta, \sin\theta)$

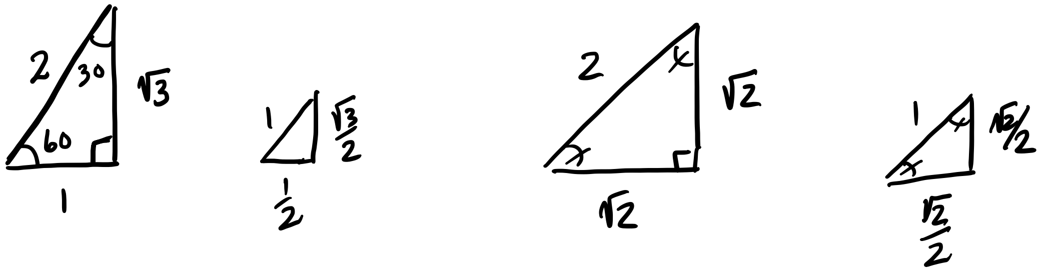
$\theta = s$ if θ is in radians

$\cos\theta = \cos(-\theta)$

How do s-def θ def fit together?



Example 4: Draw the familiar 30-60-90 and 45-45 triangles and recall how to use them to evaluate common angles for trigonometric functions.

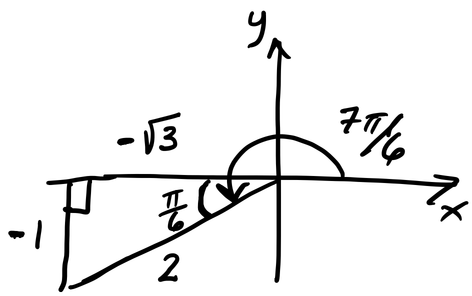


$30^\circ = \pi/6$ rad

$60^\circ = \pi/3$ rad

$45^\circ = \pi/4$ rad

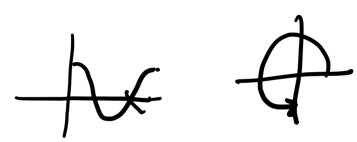
Find $\sin\theta$ and $\cos\theta$ for $\theta = \frac{7\pi}{6} = \pi + \frac{\pi}{6}$



$\sin(\frac{7\pi}{6}) = \frac{-1}{2}$

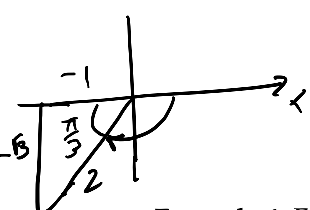
$\cos(7\pi/6) = \frac{-\sqrt{3}}{2}$

- $\sin\theta, \cos\theta, \tan\theta \quad \theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$

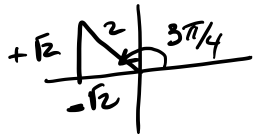


Example 5: Evaluate the following without the use of a calculator.

(a) $\sin(-\frac{2\pi}{3}) = \frac{-\sqrt{3}}{2}$ (b) $\cos(\frac{11\pi}{4}) = -\sqrt{2}/2$ (c) $\cos(\frac{3\pi}{2}) = 0$

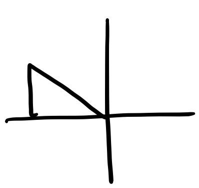


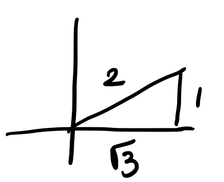
$\frac{11\pi}{4} = \frac{8\pi}{4} + \frac{3\pi}{4}$



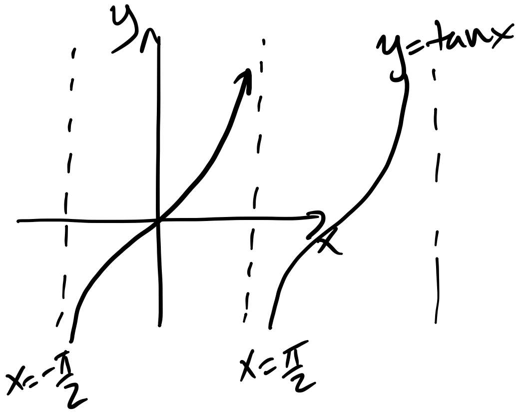
Example 6: Find the following values.

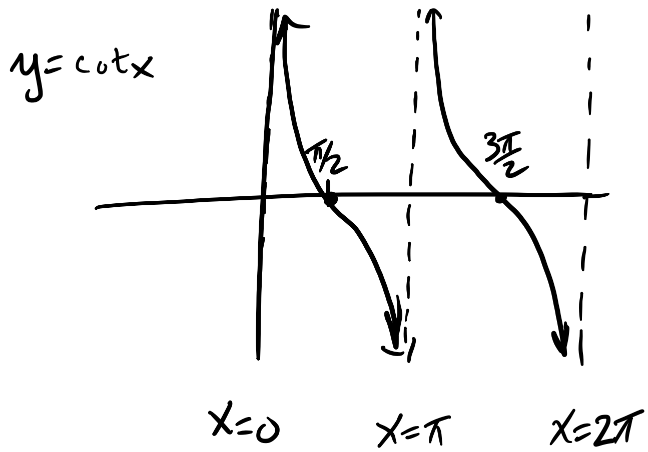
(a) $\tan(\frac{3\pi}{4}) = -1$ (b) $\cot(\frac{\pi}{6}) = \frac{a}{o} = \frac{\sqrt{3}}{1} = \sqrt{3}$ (c) $\sec(\pi) = \frac{1}{\cos\pi} = \frac{1}{-1} = -1$

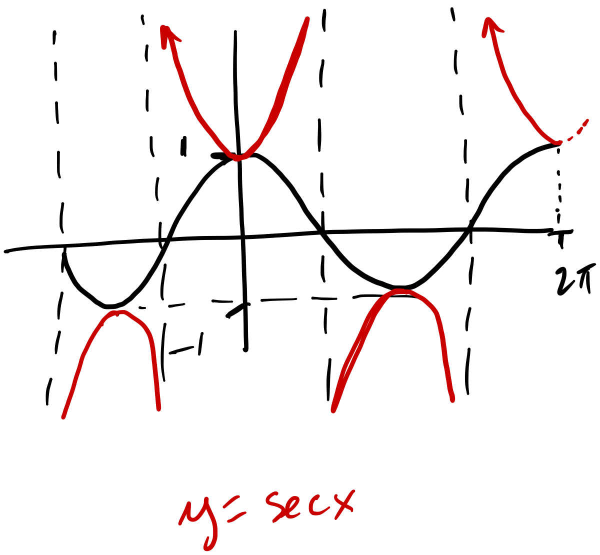


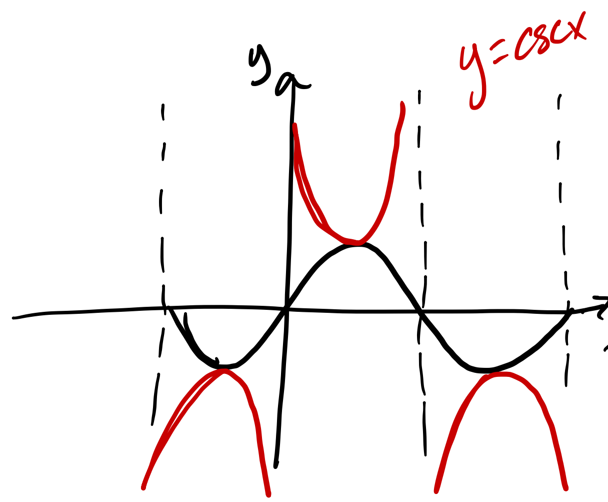


Example 7: In the space below without the use of a calculator, sketch (and label) $y = \tan x$, $y = \cot x$, $y = \sec x$, $y = \csc x$.









The Pythagorean Identities:

1. Explain *why* we know $\sin^2\theta + \cos^2\theta = 1$.

Circle definition

2. Show how to get the other two Pythagorean Identities from the one above!

$$\frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$$

$$1+\cot^2\theta=\csc^2\theta$$

$$\frac{\sin^2\theta}{\cos^2\theta}+\frac{\cos^2\theta}{\cos^2\theta}=\frac{1}{\cos^2\theta}$$

$$\tan^2\theta+1=\sec^2\theta$$