2-3 EXAMPLES

1. Evaluate each limit below. Show your work or explain your reasoning.

   (a) $\lim_{x\to 8}(1+\sqrt[3]{x})(2-x^2)$ $=(1+\sqrt[3]{8})(2-8^2) = (1+2)(-62)$
   $= 3(-62) = -186$

   (b) $\lim_{x\to -3}\frac{x^2+3x}{x^2-x-12}$ $= \lim_{x\to -3}\frac{x(x+3)}{(x-4)(x+3)} = \lim_{x\to -3}\frac{x}{x-4} = \frac{-3}{-3-4} = \frac{3}{7}$

   (c) $\lim_{x\to 4}\frac{x^2}{x^2-x-12}$ $=$ DNE. $\lim_{x\to 4^+}\frac{x^2}{(x-4)(x+3)} = +\infty$
   $\lim_{x\to 4^-}\frac{x^2}{(x-4)(x+3)} = -\infty$

   (d) $\lim_{x\to -3}\frac{\frac{1}{3}+\frac{1}{x}}{x+3}$ $= \lim_{x\to -3}\frac{\frac{x+3}{3x}}{x+3} = \lim_{x\to -3}\frac{1}{3x} = -\frac{1}{9}$

   (e) $\lim_{x\to 0}\frac{|x|}{x}$ $=$ DNE. $\lim_{x\to 0^+}\frac{|x|}{x} = \lim_{x\to 0^+}\frac{x}{x} = 1.$
   $\lim_{x\to 0^-}\frac{|x|}{x} = \lim_{x\to 0^-}\frac{-x}{x} = -1.$

   (f) $\lim_{x\to 5^-}\frac{3x-15}{|5-x|}$ $= \lim_{x\to 5^-}\frac{3(x-5)}{5-x} = \lim_{x\to 5^-}\frac{3(x-5)}{-(x-5)} = -3$

   Since $x\to 5^-$, $5-x>0$.
   So $|5-x| = 5-x$

   (g) $\lim_{x\to \pi}\frac{2x}{\tan^2 x}$ $= +\infty$

   As $x\to\pi$, $2x\to 2\pi$ (positive, nonzero) and $\tan x\to 0$. We know $\tan^2 x \geq 0$ always.

2. Give an example of a polynomial:

3. Give an example of a rational function:

4. Give an example of a function that is not a rational function:

5. Is it fair to assume $\lim_{x \to a} f(x) = f(a)$? Why or why not?

6. What if you assume $f(x)$ is a *rational function*?

7. What if you assume $f(x)$ is a *polynomial*?