2-5 EXAMPLES

1. State the definition of what it means for a function $f(x)$ to be continuous at $x = c$.

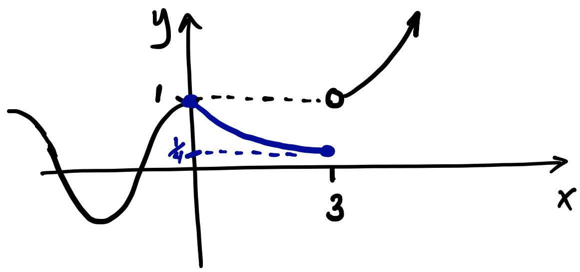
$f(x)$ is continuous if ① $f(c)$ exists,

② $\lim_{x \to c} f(x)$ exists, and

③ $\lim_{x \to c} f(x) = f(c)$

2. Given $h(x) = \begin{cases} \cos x & x < 0 \\ \frac{1}{x+1} & 0 \le x \le 3 \\ e^{x-3} & 3 < x \end{cases}$

(a) Sketch $h(x)$.



(b) Use the definition to *show* whether or not h is continuous at $x = 0$.

h is continuous at $x=0$.

① $h(0)=1$

② $\lim_{x\to 0^-} h(x) = \lim_{x \to 0^-} \cos 0 = 1$; $\lim_{x \to 0^+} h(x) = \lim_{x\to 0^+} \frac{1}{x+1} = \frac{1}{0+1} = 1$. So $\lim_{x\to 0} h(x) = 1$.

③ $h(0) = 1 = \lim_{x \to 0} h(0)$

(c) Use the definition to *show* whether or not h is continuous at $x = 1$.

h is continuous at $x=1$.

① $h(1) = \frac{1}{2}$

② $\lim_{x\to 1} h(x) = \lim_{x \to 1} \frac{1}{x+1} = \frac{1}{2}$

③ $\lim_{x \to 1} h(x) = \frac{1}{2} = h(1)$.

(d) Use the definition to *show* whether or not h is continuous at $x = 3$.

h is not continuous at $x=3$.

$\lim_{x\to 3^-} h(x) = \frac{1}{x+1} = \frac{1}{4}$; $\lim_{x \to 3^+} h(x) = \lim_{x\to 3^+} e^{x-3} = e^0 = 1$;

Since the two onesided limits are different, the $\lim_{x\to 3} h(x)$ does not exist. Since the limit does not exist at $x=3$, the graph is not continuous at $x=3$.

3. Use the Intermediate Value Theorem to show that there must be some x value such that $f(x) = x - \ln x = 10$.

Since $f(x) = x - \ln x$ is continuous for all $x > 0$, the Intermediate Value Theorem applies.

Observe:

$$f(1) = 1 - \ln 1 = 1 < 10 \quad \text{and}$$

$$f(e^3) = e^3 - \ln e^3 = e^3 - 3 > 10.$$

So there is some c in $(1, e^3)$ so that $f(c) = 10$.