1. Write two expressions for the slope of the tangent line to the function f(x) at x=a. (Hint: Both should involve a limit. One has $x\to a$ and one has $h\to 0$. Try to produce the expressions by thinking about where they come from.)

$$m_{+an} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

For all of the problems on this worksheet, you need to use one of the expressions above. You should NOT use a short-cut rule we have not yet covered.

2. For each function and a-value below, i) find the slope of the tangent line to f(x) at x = a and (ii) write the equation of the line tangent to f(x) at x = a.

(a)
$$f(x) = \sqrt{2x}$$
 when $a = 8$.

(i)
$$m = \lim_{h \to 0} \frac{\sqrt{2(8+h)} - \sqrt{2.8}}{h} = \lim_{h \to 0} \frac{\sqrt{16+2h} - 4}{h} = \lim_{h \to 0} \frac{\sqrt{16+2h} + 4}{\sqrt{16+2h} + 4}$$

$$= \lim_{h \to 0} \frac{16+2h-16}{h(\sqrt{16+2h} + 4)} = \lim_{h \to 0} \frac{2}{h(\sqrt{16+2h} + 4)} = \lim_{h \to 0} \frac{2}{\sqrt{16+2h} + 4} = \frac{2}{8} = \frac{1}{4}$$

(b)
$$f(x) = \frac{3}{2-x}$$
 when $a = 1$.

(i) $m = \lim_{h \to 0} \frac{1}{h} \left[\frac{3}{2-(1+h)} - \frac{3}{2-1} \right] = \lim_{h \to 0} \frac{1}{h} \left[\frac{3}{1-h} - \frac{3}{1} \right] = \lim_{h \to 0} \frac{1}{h} \left[\frac{3-3(h)}{1-h} \right] = \lim_{h \to 0} \frac{1}{h} \left[\frac{3-3(h)}{1-h} \right] = \lim_{h \to 0} \frac{1}{h} \left[\frac{3h}{1-h} \right] = \lim_{h \to 0} \frac{3}{1-h} = 3$

line:
$$y-3=3(x-1)$$
 or $y=3x$

3. (a) For $f(x) = 2x - x^2$, find f'(a).

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{2(a+h) - (a+h)^2 - [2a - a^2]}{h}$$

$$= \lim_{h \to 0} \frac{2a+2h-a^2-2ah-h^2-2a+a}{h} = \lim_{h \to 0} \frac{2h-2ah-h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2-2a-h)}{h} = \lim_{h \to 0} \frac{2-2a-h}{h} = 2-2a$$

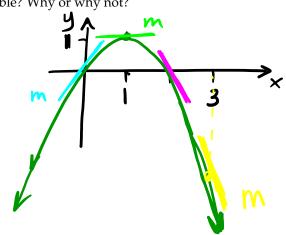
answer: f'(a) = 2-2a

(b) Find f'(0), f'(1), f'(2) and f'(3). (You could just make a table of values...)

(c) Do your answers to part (b) seem reasonable? Why or why not?

$$f(x) = 2x - x^2 = x(2-x)$$

Do these seem reasonable?
Well, yes. The numbers fit with what I know the graph looks like ...



2