1. State the definition of the derivative of the function $f(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

For all of the problems on this worksheet, you need to use the definition above. You should NOT use a short-cut rule we have not yet covered.

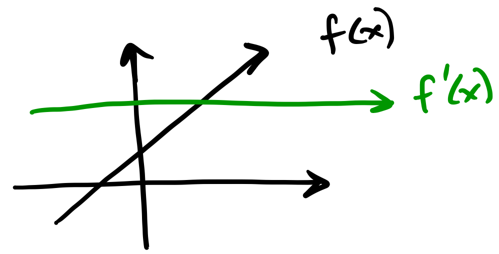
2. For each function (i) find $f'(x)$ using the definition, (ii) graph $f(x)$ and $f'(x)$ on the same axes, and (iii) state their domains.

   (a) $f(x) = mx + b$ where m and b are fixed constants.

(i) $$f'(x) = \lim_{h\to 0} \frac{(m(x+h)+b)-(mx+b)}{h} = \lim_{h\to 0} \frac{mx+mh+b-mx-b}{h}$$

$$= \lim_{h\to 0} \frac{mh}{h} = \lim_{h\to 0} m = m. \quad \boxed{f'(x)=m}$$

(ii)



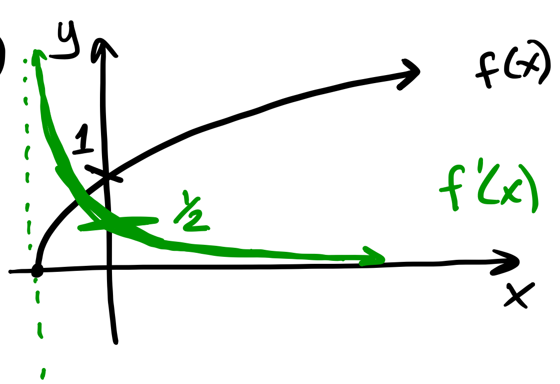
domain of both is $\mathbb{R}$.

   (b) $f(x) = \sqrt{x+1}$

(i) $$f'(x) = \lim_{x\to 0} \frac{\sqrt{x+h+1}-\sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1}+\sqrt{x+1}}{\sqrt{x+h+1}+\sqrt{x+1}} = \lim_{x\to 0} \frac{x+h+1-x-1}{h(\sqrt{x+h+1}+\sqrt{x+1})}$$

$$= \lim_{x\to 0} \frac{h}{h(\sqrt{x+h+1}+\sqrt{x+1})} = \lim_{x\to 0} \frac{1}{\sqrt{x+h+1}+\sqrt{x+1}} = \frac{1}{2\sqrt{x+1}}. \quad \text{So } f'(x) = \frac{1}{2}(x+1)^{-1/2}$$

(ii)



domain of $f(x)$: $[0, \infty)$
domain of $f'(x) = (0, \infty)$

(c) $f(x) = |x|$ $\qquad f'(x) = \lim_{h \to 0} \frac{|x+h| - |x|}{h}$

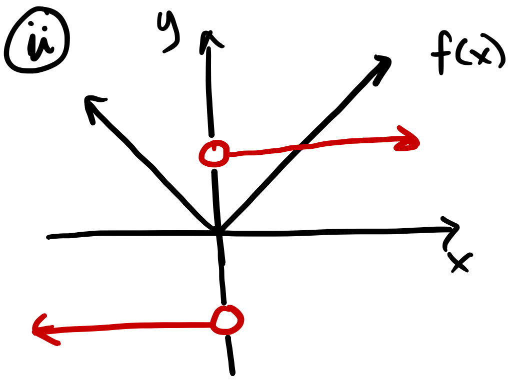
(i) If $x > 0$, then $f'(x) = \lim_{h \to 0} \frac{|x+h| - |x|}{h} = \lim_{h \to 0} \frac{x+h-x}{h} = \lim_{h \to 0} 1 = 1$

If $x < 0$, then $f'(x) = \lim_{h \to 0} \frac{|x+h| - |x|}{h} = \lim_{h \to 0} \frac{-(x+h)+x}{h} = \lim_{h \to 0} -1 = -1$

$x+h<0$

so $|x+h| = -(x+h)$

(ii)



domain $f(x)$ is $\mathbb{R}$

domain of $f'(x)$ is $(-\infty, 0) \cup (0, \infty)$

3. For each function below, sketch its derivative on the same set of axes.