Let $f(x) = \sqrt{x}$.

1. Find the equation of the line tangent to the curve of $f(x)$ at $x = 4$.

$f(x) = x^{1/2}$ $\qquad f(4) = 2$ $\qquad y - 2 = \frac{1}{4}(x-4)$

$f'(x) = \frac{1}{2}x^{-1/2}$ $\qquad f'(4) = \frac{1}{4}$

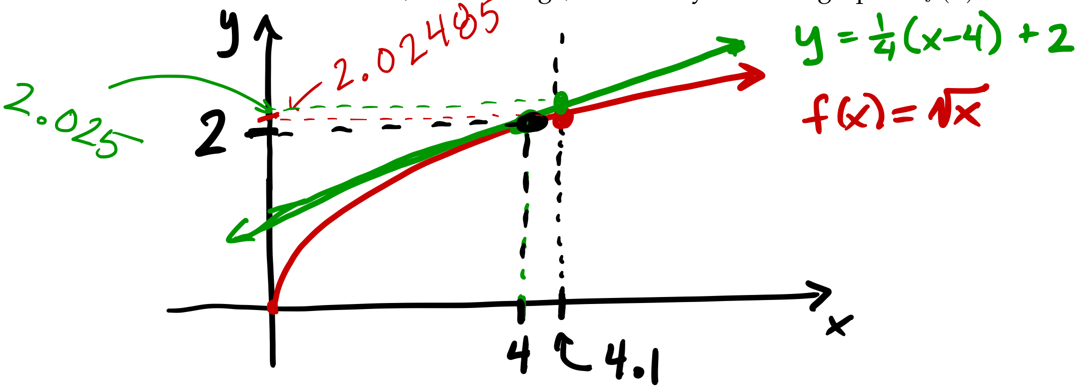
or

$y = \frac{1}{4}(x-4) + 2$

or

$y = \frac{1}{4}x + 1$

2. On the same set of axes, draw a large, reasonably accurate graph of $f(x)$ and its tangent line. Label them.



3. Correct to at least 5 decimal places, find the y-value of the function $f(x)$ when $x = 4.1$ and find the y-value of the tangent line when $x = 4.1$. Graph and label these points on the axes above.

$f(4.1) = \sqrt{4.1} = 2.02485$

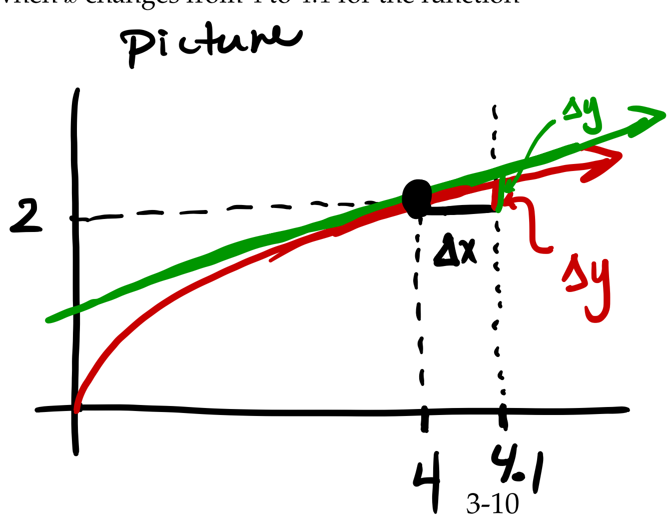
$y(4.1) = \frac{1}{4}(4.1-4)+2 = (0.25)(0.1)+2 = 2.025$

4. Correct to at least 3 decimal places, determine the change in y when x changes from 4 to 4.1 for the function $f(x)$ and for the tangent line. Sketch these quantities.

x changes from 4 to 4.1

f(x) changes from 2 to 2.02485
(So change in y is 0.02485)

y changes from 2 to 2.025.
(so change in y is 0.025)



Practice Problems (round 1)

1. (a) Without the use of a calculator, find the linear approximation of $f(x) = \sin x$ at $x = 0$ and use it to approximate $\sin(0.1)$.

$f(x) = \sin x$; $f(0) = \sin 0 = 0$

$f'(x) = \cos x$, $f'(0) = 1$

$y - 0 = 1 \cdot (x - 0)$

$y = x$

$\boxed{L(x) = x}$

$\boxed{\sin(0.1) \approx L(0.1) = 0.1}$

(b) Use a calculator to find $\sin(0.15)$ exactly and compare to your approximation.

$f(0.1) = \sin(0.1) = 0.0998334$

$L(0.1) = 0.1$

$f(0.1) - L(0.1) = -0.0001666$

2. (a) Find the differential for $y = x^2 - 4x$.

$dy = (2x - 4)\, dx$

(b) Use the differential to estimate Δy when $x = 3$ and $\Delta x = dx = 0.5$. (Don't use a calculator!)

$\Delta y \approx dy = (2 \cdot 3 - 4)(0.5) = 2(.5) = 1$

(c) Now use a calculator to find Δy precisely and compare.

$f(3.5) - f(3) = \left[(3.5)^2 - 4(3.5)\right] - \left[3^2 - 4 \cdot 3\right]$

$= 1.25$

close.

Let $f(x) = \sqrt{x}$.

1. Find the **linear approximation** of $f(x)$ at $x = 4$. [Replace y with $L(x)$.]

$f(x) = x^{1/2}$ $\quad f(4) = 2$ $\quad y - 2 = \boxed{\frac{1}{4}(x-4)}$

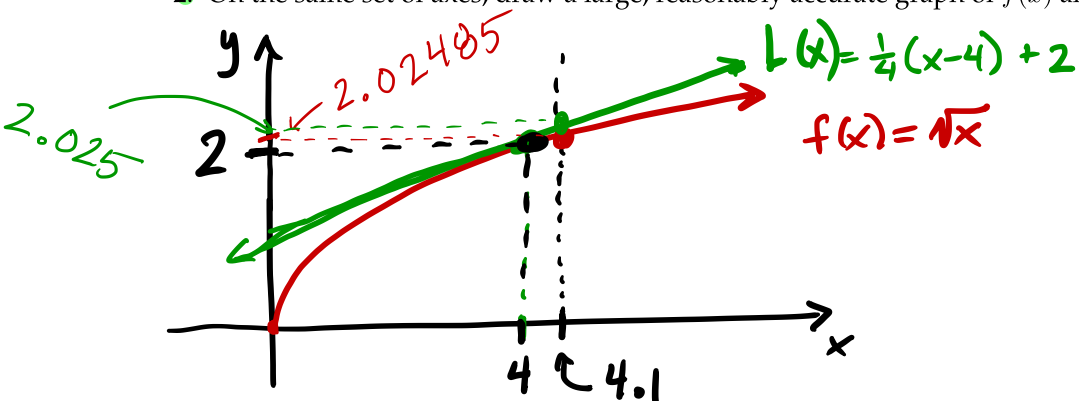
$f'(x) = \frac{1}{2}x^{-1/2}$ $\quad f'(4) = \frac{1}{4}$

or

$y = \frac{1}{4}(x-4) + 2$

Only difference $\quad \boxed{L(x) = \frac{1}{4}(x-4)+2}$

2. On the same set of axes, draw a large, reasonably accurate graph of $f(x)$ and ~~its tangent line~~ the linear approximation of f(x). Label them.



3. **Use the linearization of** $f(x)$ **at** $x = 4$ **to estimate** $f(4.1)$. **How good is this estimation?**

$f(4.1) = \sqrt{4.1} = 2.02485$

$L(4.1) = \frac{1}{4}(4.1-4)+2 = (0.25)(0.1)+2 = 2.025$

How good is the estimation? Excellent. $f(4.1) - L(4.1) \le 0.0002$

4. **Use the differential to estimate** Δy **when** $x = 4$ **and** $\Delta x = dx = 0.1$.

$dy = \frac{1}{2}x^{-1/2}\,dx$ ← the differential of f(x)

$\Delta y \approx dy = \frac{1}{2}(4)^{-1/2}(0.1) = 0.25$

↑ where have we seen this calculation before?

Practice Problems (round 2)

1. Use a linear approximation to estimate $\sqrt[3]{124}$

Choose the function... $f(x) = \sqrt[3]{x} = x^{1/3}$

Choose the point of tangency $x = 125$

$f'(x) = \frac{1}{3}x^{-2/3}$; $f'(125) = \frac{1}{3} \cdot \frac{1}{25} = \frac{1}{75}$

$\frac{1}{75} = \frac{1}{3} \cdot \frac{1}{25}$

$(0.3 \cdot 0.04)$

$= 0.01\overline{3}$

$f(125) = 5$

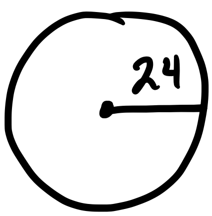
$y - 5 = \frac{1}{75}(x - 125)$

$L(x) = 5 + \frac{x-125}{75}$

$\boxed{\begin{aligned} &\underline{\text{Estimation}}: \\ &L(124) = 5 + \frac{124-125}{75} \\ &\quad = 5 - \frac{1}{75} = 4.98\overline{6} \end{aligned}}$

aside: $\sqrt[3]{124} = 4.98663$

2. The radius of a circular disk is given at 24 cm with a maximum error in measurement of at most 0.2 cm. Use differentials to estimate the maximum error in the calculated area of the disc. Does this error seem large?



$24 = r$ where $\Delta r \le 0.2$

$A = \pi r^2$

$dA = 2\pi r\, dr$

$\Delta A \approx dA = 2\pi(24)(0.2) = 9.6\pi \approx 30.2 \text{ cm}^2.$

Large? Kinda... but...

$A = \pi(24)^2 = 576\pi \approx 1810 \text{ cm}^2.$

So relative error $= \frac{\Delta A}{A} \approx \frac{30}{1810} = 0.017$ or 1.7%.