3-2 WARM-UP

1. Fill in the blanks below:

(a) $\frac{d}{dx}\left[f(x)\cdot g(x)\right] =$ $f'\cdot g + f\cdot g'$

(b) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] =$ $\frac{g\cdot f' - f\cdot g'}{g^2}$

2. Find the derivatives for each function below and compare your methods:

(a) $f(x) = \frac{20}{\sqrt[3]{x}}$ $= 2x^{-1/3}$

$f'(x) = 2\left(\frac{-1}{3}\right)x^{-4/3}$

$= -\frac{2}{3}x^{-4/3}$

(just power rule)

(b) $f(x) = \frac{20}{x^2+20}$

$f' = \frac{(x^2+20)\cdot 0 - 20(2x)}{(x^2+20)^2}$

$= \frac{40x}{(x^2+20)^2}$

(requires quotient rule!)

3. Find the derivatives for each function below and compare your methods:

(a) $f(x) = 20\left(\frac{x-x^3}{x^{3/5}}\right)$

$f(x) = 20(x-x^3)(x^{-3/5})$

$= 20(x^{2/5} - x^{12/5})$;

$f'(x) = 20\left[\frac{2}{5}x^{-3/5} - \frac{12}{5}x^{7/5}\right]$

(b) $f(x) = e^x\left(\frac{x-x^3}{x^{3/5}}\right) = e^x(x^{2/5} - x^{12/5})$.

$f'(x) = e^x(x^{2/5} - x^{12/5}) + e^x\left(\frac{2}{5}x^{-3/5} - \frac{12}{5}x^{7/5}\right)$

$= e^x\left[x^{2/5} - x^{12/5} + \frac{2}{5}x^{-3/5} - \frac{12}{5}x^{7/5}\right]$

4. Find the derivative of $f(x) = \frac{x^2+1}{xe^x}$ (Use Quotient Rule and Product Rule.)

$$f'(x) = \frac{xe^x(2x) - (x^2+1)\frac{d}{dx}[xe^x]}{(xe^x)^2}$$

$$= \frac{2x^2e^x - (x^2+1)[x\cdot e^x + 1\cdot e^x]}{(xe^x)^2}$$

$$= \frac{e^x[2x^2 - (x^2+1)(x+1)]}{(xe^x)^2}$$

$$= \frac{e^x[2x^2 - x^3 - x^2 - x - 1]}{(xe^x)^2} = \frac{x^2 - x^3 - x - 1}{x^2e^x}$$

5. Assume $s(t) = 3te^t$ gives the position of an object where s is measured in feet and t is measured in seconds. Find $s'(1)$ and $s''(1)$ and interpret your answers.

$$s'(t) = 3\cdot e^t + 3te^t = 3e^t(1+t)$$

$$s''(t) = 3e^t(1+t) + 3e^t\cdot 1 = 3e^t(t+2)$$

$s'(1) = 3e(1+1) = 6e$ ft/sec. This is the velocity of the object at 1 second.

$s''(1) = 3\cdot e\cdot(1+2) = 9e$ ft/sec^2. This is the acceleration of the object at 1 second.