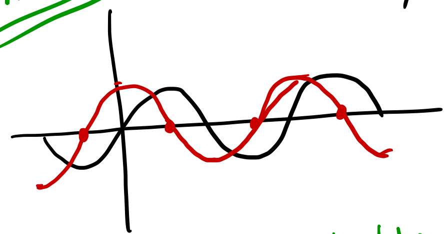
# 3-3 Day 2

1. Using the fact that $\frac{d}{dx}[\sin x] = \cos x$ and the geometric relationship between $f(x) = \sin x$ and $g(x) = \cos x$, explain why

Know

$f(x) = \sin x$, $f' = \cos x$.

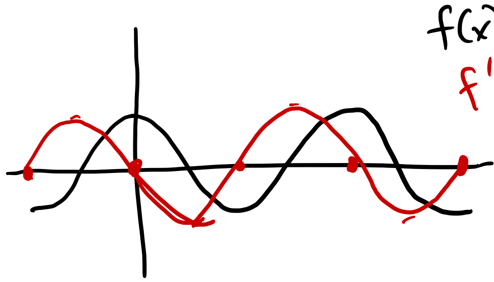
$$\frac{d}{dx}[\cos x] = -\sin x.$$



shift the picture left

$f(x) = \cos x$

$f'(x) = -\sin x$



$\cos x$ is $\sin x$ shifted right by $\pi/2$ units.
So the derivative of $\cos x$ ought to be the derivative of $\sin x$ shifted by $\pi/2$ units

2. Show $\frac{d}{dx}[\tan x] = \sec^2 x$ using the Quotient Rule and the derivatives of sine and cosine.

$$y = \tan x = \frac{\sin x}{\cos x}$$

$$y' = \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

**Derivatives of Trigonometric Functions:**

- $\frac{d}{dx}(\sin x) =$ $\cos x$
- $\frac{d}{dx}(\cos x) =$ $-\sin x$
- $\frac{d}{dx}(\tan x) =$ $\sec^2 x$
- $\frac{d}{dx}(\csc x) =$ $-\csc x \cot x$
- $\frac{d}{dx}(\sec x) =$ $\tan x \sec x$
- $\frac{d}{dx}(\cot x) =$ $-\csc^2 x$

3. Find the derivatives of each of the following:

   (a) $y = e^x(\tan x - \sec x)$

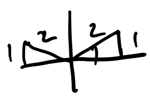
$$y' = e^x(\tan x - \sec x) + e^x(\sec^2 x - \sec x \tan x)$$
$$= e^x[\tan x - \sec x + \sec^2 x - \sec x \tan x]$$

   (b) $g(\theta) = \frac{\sin\theta}{\cos\theta + 1}$

$$g'(\theta) = \frac{(\cos\theta + 1)(\cos\theta) - \sin\theta(-\sin\theta)}{(\cos\theta+1)^2} = \frac{\cos^2\theta + \cos\theta + \sin^2\theta}{(\cos\theta+1)^2} = \frac{\cos\theta + 1}{(\cos\theta+1)^2}$$
$$= \frac{1}{\cos\theta + 1}$$

4. For what values of $t$ does the graph of $f(t) = t + 2\cos t$ have a horizontal tangent?

$$f'(t) = 1 - 2\sin t = 0$$
$$\sin t = \frac{1}{2}$$



$$t = \frac{\pi}{6} + 2\pi k \quad \text{or} \quad \frac{5\pi}{6} + 2\pi k$$

5. An elastic band is hung on a hook and a mass is hung on the lower end of the band. When the mass is pulled down 2 cm past its rest position and then released, it vibrates vertically. The equation of motion is

$$s = 2\cos t + 3\sin t, \quad \text{for } t \geq 0,$$

   where $s$ is measured in centimeters and $t$ is measured in seconds. (We are taking the positive direction to be downward.)

   (a) Find $s(0)$, $s'(0)$, and $s''(0)$ including units.

$$s'(t) = -2\sin t + 3\cos t$$
$$s''(t) = -2\cos t - 3\sin t$$

$$s(0) = 2 \text{ cm}$$
$$s'(0) = 3 \text{ cm/s}$$
$$s''(0) = -2 \text{ cm/s}^2$$

   (b) What do your answers from part (a) tell you about the mass? Do your answers make sense?

$s(0) = 2$ confirms the object starts 2 cm below equilibrium.

$s'(0) = 3$ cm/s tells us that the object is released with downward velocity.

$s''(0) = -2$ cm/$s^2$ confirms that the spring is pulling up on the object and slowing the object down.