3-3 START UP

1. Use your calculator to evaluate the limits below:

   (a) $\lim_{x\to 0} \frac{\sin x}{x}$ assuming x is in radians

   $$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

   (b) $\lim_{x\to 0} \frac{\sin x}{x}$ assuming x is in degrees

   $$\lim_{x\to 0} \frac{\sin x}{x} = 0.01745$$

   (c) $\lim_{x\to 0} \frac{\cos x - 1}{x}$ assuming x is in radians

   $$\lim_{x\to 0} \frac{\cos x - 1}{x} = 0$$

2. You are going to establish the derivative of $f(x) = \sin x$.

   (a) Start by applying the definition of the derivative to $f(x) = \sin x$. (The definition with the limit and the little h.)

   $$\lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

   (b) There is a trig identity: $\sin(a+b) = \sin a \cos b + \sin b \cos a$. Use this identity to rewrite the term $\sin(x+h)$.

   $$\lim_{h\to 0} \frac{\sin x \cosh + \sinh \cos x - \sin x}{h}$$

   (c) Collect the terms with $\sin x$ together and any terms with $\cos x$. Once you have done this, see if you can use the limits from part 1 to evaluate the limit and find the derivative of $\sin x$.

   $$\lim_{h\to 0} \left[\sin x\left(\frac{\cosh - 1}{h}\right) + \left(\frac{\sin h}{h}\right)\cos x\right]$$

   $$= (\sin x)\left[\lim_{h\to 0} \frac{\cosh - 1}{h}\right] + (\cos x)\left[\lim_{h\to 0} \frac{\sin h}{h}\right]$$

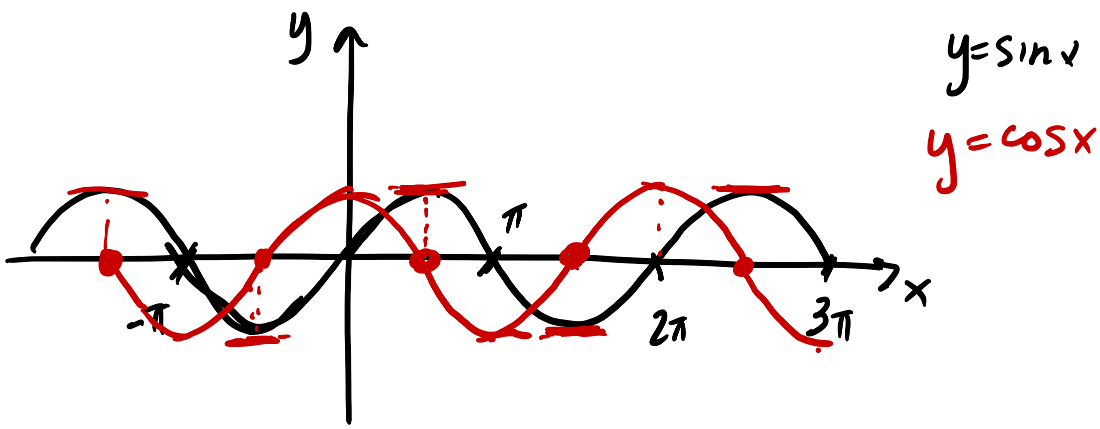
   $$= (\sin x)\ (0) \quad + (\cos x)\cdot(1) \quad = \cos x$$

3. Does it matter whether you use degrees or radians to find the derivative of $f(x) = \sin x$?

apparently. See yellow arrow

The 1 would be 0.01745...

4. For $f(x) = \sin x$, sketch $f(x)$ and $f'(x)$ on the same axes and check the "reasonableness" of your answer.



Where y=sin x has a horizontal tangent, y=cos x is zero.

5. Use the fact that you know the derivative of $y = \sin x$ to find the derivative of $y = \csc x$.

$$y = \csc x = \frac{1}{\sin x}$$

Apply Quotient Rule:

$$y' = \frac{(\sin x)\cdot 0 - 1\cdot \cos x}{\sin^2 x} = \frac{-\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}$$

$$= -\cot x \csc x$$