3-4 Day 1

1. Find the derivative of the function.

   (a) $g(x) = (2 + 3x - x^2)^5$

$$g'(x) = 5(2+3x-x^2)^4 \cdot (3-2x)$$
$$= 5(3-2x)(2+3x-x^2)^4$$

   (b) $h(x) = \frac{30}{\sqrt[3]{4x-5}}$ $= 30(4x-5)^{-1/3}$

$$h'(x) = 30 \cdot \left(\frac{-1}{3}\right)(4x-5)^{-4/3} \cdot 4$$
$$= -40(4x-5)^{-4/3}$$

   (c) $f(t) = e^{t-t^2}$

$$f'(t) = e^{t-t^2} \cdot (1-2t) = (1-2t)e^{t-t^2}$$

   (d) $y = e^{ax}\cos(bx)$, where a and b are fixed constants

$$\frac{dy}{dx} = e^{ax}\left[\frac{d}{dx}(\cos bx)\right] + \frac{d}{dx}\left[e^{ax}\right] \cdot \cos(bx)$$
$$= e^{ax}\left[-\sin(bx) \cdot b\right] + \left[ae^{ax}\right] \cdot \cos(bx)$$
$$= e^{ax}\left[-b\sin(bx) + a\cos(bx)\right]$$

   (e) $f(x) = \sqrt{\frac{2x}{x-1}}$ $= (2x)^{1/2}(x-1)^{-1/2}$

$$f'(x) = (2x)^{1/2} \cdot \frac{d}{dx}\left[(x-1)^{-1/2}\right] + \frac{d}{dx}\left[(2x)^{1/2}\right] \cdot (x-1)^{-1/2}$$
$$= (2x)^{1/2} \cdot -\frac{1}{2} \cdot (x-1)^{-3/2} \cdot 1 + \frac{1}{2}(2x)^{-1/2} \cdot 2 \cdot (x-1)^{-1/2}$$

$-2x+2x-2$

$$= \frac{-(2x)^{1/2}}{2(x-1)^{3/2}} + \frac{1}{[2x(x-1)]^{1/2}} = \frac{-(2x)+2(x-1)}{2 \cdot (2x)^{1/2}(x-1)^{3/2}} = \frac{-1}{(2x)^{1/2}(x-1)^{3/2}}$$

2. (a) The volume of a snowball of radius r is $V(r) = (4/3)\pi r^3$, where r is measured in inches and V is in measured in inches cubed. Explain what $V'(2) \approx 50.265$ means in language your parents could understand.

We know intuitively that as the radius increases, the volume of the snowball increases. $V'(2) \approx 50.265$ means that when the radius is 2 inches, the rate of increase of volume relative to an increase in radius is $50.265\ in^3/in$.

(b) If you increase the radius of a snowball from 2 inches to 2.02 inches, estimate the change in volume of the snowball.

$$\Delta V \approx \frac{dV}{dr} \cdot \Delta r \approx 50.265(0.02) = 1.0053\ in^3$$

3. Under certain circumstances a rumor spreads according to the equation

$$p(t) = \frac{1}{1 + ae^{-kt}} = (1+ae^{-kt})^{-1}$$

where $p(t)$ is the proportion of the population that has heard the rumor at time t and a and k are positive constants.

(a) Find $\lim_{t\to\infty} p(t)$ and interpret your answer.

as $t \to \infty$, $e^{-kt} \to 0$. So $\lim_{t\to\infty} \frac{1}{1+ae^{-kt}} = \frac{1}{1+a\cdot 0} = 1.$

As time goes on, the proportion of population that has heard the rumor approaches the whole population. (or 1)

(b) Find the rate of spread of the rumor.

$$p'(t) = -1(1+ae^{-kt})^{-2} \cdot \frac{d}{dt}\left[1+ae^{-kt}\right] = \frac{-1}{(1+ae^{-kt})^2}(ae^{-kt}\cdot(-k))$$

$$= \frac{ake^{-kt}}{(1+ae^{-kt})^2}$$

(c) Find and interpret $p(0)$ and $p'(0)$.

$p(0) = \frac{1}{1+ae^0} = \frac{1}{1+a}$; the proportion of the population initially aware of the rumor.

$p'(0) = \frac{ake^0}{(1+ae^0)^2} = \frac{ak}{(1+a)^2}$; the rate at which the rumor is spreading initially (when $t=0$) with respect to time.