3-4 Day 2

1. A rocket is launching, and its height h in meters is a function of t in seconds (so we are considering the function $h(t)$). Explain what $h'(10) = 1035$ means in language your mom could understand. You answer must include units.

Ten seconds after launch, the rocket's height is increasing at a rate of 1035 m/s.

2. Find the derivative of the function.

(a) $f(x) = xe^{1/x}$ $= x e^{x^{-1}}$

$f'(x) = 1 \cdot e^{x^{-1}} + x \cdot e^{x^{-1}} \cdot (-1x^{-2})$

$= e^{x^{-1}} - x^{-1} e^{x^{-1}}$

(b) $g(x) = \frac{tan(2x)}{1+x}$ $= \tan(2x)(1+x)^{-1}$

$g'(x) = [\sec^2(2x) \cdot 2](1+x)^{-1} + (\tan 2x)[-1(1+x)^{-2} \cdot 1]$

$= \frac{2\sec^2(2x)}{1+x} - \frac{\tan 2x}{(1+x)^2}$

(c) $y = (1 + x^2)e^x \sec x$

$y' = 2x \cdot e^x \sec x + (1+x^2)\frac{d}{dx}[e^x \sec x]$

$= 2xe^x \sec x + (1+x^2)[e^x \cdot \sec x + e^x \cdot \sec x \tan x]$

$= e^x \sec x(2x + (1+x^2)(1+\tan x))$

(d) $h(x) = \sin(5x - e^{-5x})$

$h'(x) = \cos(5x - e^{-5x})[5 - e^{-5x}(-5)]$

$= 5(1+e^{-5x})\cos(5x - e^{-5x})$

(e) $f(x) = \sqrt{1 + xe^{-2x}}$ $= (1+xe^{-2x})^{1/2}$

$f'(x) = \frac{1}{2}(1+xe^{-2x})^{-1/2} \cdot \frac{d}{dx}[1+xe^{-2x}]$

$= \frac{1}{2}(1+xe^{-2x})^{-1/2}(1 \cdot e^{-2x} + x \cdot e^{-2x} \cdot -2)$

$= \frac{1}{2}e^{-2x}(1-2x)(1+xe^{-2x})^{-1/2}$

3. Find the equation of the line tangent to the graph of $f(x) = \sqrt{1+x^3}$ when $x = 2$.

$$f(2) = \sqrt{1+2^3} = 3; \quad \text{point } (2,3)$$

$$f'(x) = \frac{1}{2}(1+x^3)^{-1/2}(3x) = \frac{3x^2}{2\sqrt{1+x^3}}$$

$$f'(2) = \frac{3\cdot 4}{2\sqrt{9}} = \frac{12}{6} = 2$$

line: $y - 3 = 2(x-2)$

$$y = 2x - 4 + 3$$

$$\boxed{y = 2x - 1}$$

4. Find y'' for $y = \frac{1}{(1+\tan x)^2}$. $= (1+\tan x)^{-2}$

$$y' = -2(1+\tan x)^{-3}(\sec^2 x) = -2\underbrace{(1+\tan x)^{-3}}_{f}\underbrace{(\sec x)^2}_{g}$$

$$y'' = -2\Big[\underbrace{\left(-3(1+\tan x)^{-4}\cdot \sec^2 x\right)}_{f'}\cdot \underbrace{\sec^2 x}_{g} + \underbrace{(1+\tan x)^{-3}}_{f}\cdot \underbrace{2(\sec x)^1(\sec x \tan x)}_{g'}\Big]$$

$$= -2\left(-3\sec^4 x\,(1+\tan x)^{-4} + 2\sec^2 x \tan x\,(1+\tan x)^{-3}\right)$$

$$= \frac{-2\sec^2 x}{(1+\tan x)^4}\left[-3\sec^2 x + 2\tan x(1+\tan x)\right]$$

5. If the equation of motion of a particle is given by

$$s = A\cos(\omega t + \delta)$$

a particle is said to undergo simple harmonic motion. Find the velocity of the particle at time t and determine when the velocity is zero.

$$v(t) = s'(t) = A\left[-\sin(\omega t+\delta)\cdot\omega\right]$$

$$= -A\omega\sin(\omega t+\delta)$$

$v=0$ means $-A\omega\sin(\omega t+\delta)=0$ divide by $-A\omega$

So $\sin(\omega t+\delta)=0$

So $\omega t+\delta=\pi k$. So $t=\dfrac{\pi k-\delta}{\omega}$

6. The brightness of a star is given by

$$B(t) = 4.0 + 0.35\sin\left(\frac{2\pi t}{5.4}\right)$$

where t is measured in days. Find the rate of change of brightness after one day and interpret your answer.

$$B'(t) = 0.35\cdot\cos\left(\frac{2\pi}{5.4}t\right)\cdot\frac{d}{dt}\left[\frac{2\pi}{5.4}t\right]$$

$$= (0.35)\cos\left(\frac{2\pi}{5.4}t\right)\cdot\frac{2\pi}{5.4}$$

$$= \frac{2\pi(0.35)}{5.4}\cos\left(\frac{2\pi}{5.4}t\right).$$

$$B'(1) = \frac{2\pi(0.35)}{5.4}\cos\left(\frac{2\pi}{5.4}\right) \approx 0.1613$$

After 1 day, the brightness of the star is increasing at a rate of 0.1613 per day.