1. Find $\frac{dy}{dx}$ for each of expression below by implicit differentiation.

   (a) $e^{xy} = x + y + 1$

$$e^{xy}\cdot\left(1\cdot y + x\cdot\frac{dy}{dx}\right) = 1 + \frac{dy}{dx}$$

$$ye^{xy} + xe^{xy}\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\left(xe^{xy} - 1\right)\frac{dy}{dx} = 1 - ye^{xy}$$

$$\frac{dy}{dx} = \frac{1 - ye^{xy}}{xe^{xy} - 1}$$

   (b) $x = \sin y$

$$1 = (\cos y)\frac{dy}{dx}. \quad \text{So } \frac{dy}{dx} = \frac{1}{\cos y}$$

   (c) $x = \cos y$

$$1 = -\sin y \cdot \frac{dy}{dx}$$

$$\text{So } \frac{dy}{dx} = \frac{-1}{\sin y}$$

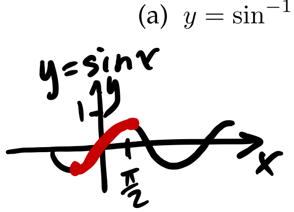
   (d) $x = \tan y$

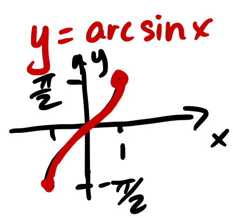
$$1 = (\sec^2 y)\cdot\frac{dy}{dx}$$

$$\text{So } \frac{dy}{dx} = \frac{1}{\sec^2 y}$$

2. For each inverse trigonometric function below, sketch its graph and state its domain and range.

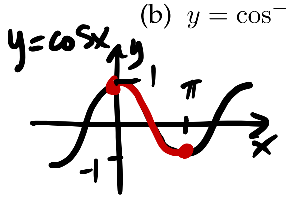
   (a) $y = \sin^{-1} x$



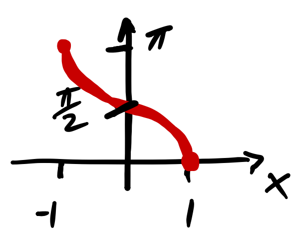


domain: $[-1, 1]$
range: $[-\pi/2, \pi/2]$

   (b) $y = \cos^{-1} x$

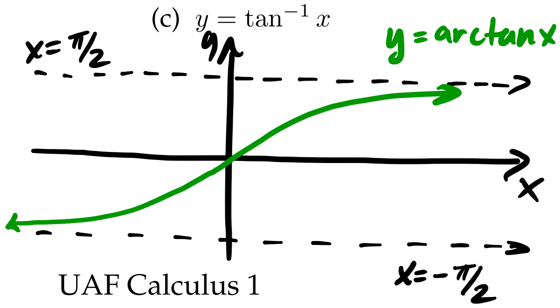


y= arccosx



domain: $[-1, 1]$
range: $[0, \pi]$

   (c) $y = \tan^{-1} x$



domain: $(-\infty, \infty)$
range: $(-\frac{\pi}{2}, \frac{\pi}{2})$

Why did we just do that?

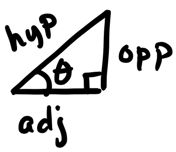
Recall: $y = \arcsin x \iff x = \sin y$

Substitute!

We found $\dfrac{dy}{dx} = \dfrac{1}{\cos y} = \dfrac{1}{\cos(\arcsin(x))} = \dfrac{1}{\sqrt{1-x^2}}$

We can do better!

Recall:

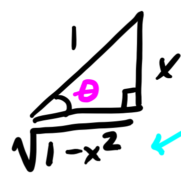


$\sin\theta = \dfrac{opp}{hyp}$. So $\arcsin\left(\dfrac{opp}{hyp}\right) = \theta$

So, $\arcsin x = \arcsin\left(\dfrac{x}{1}\right) = \theta$ where

opp

hyp



How do I know this is positive?

Thus, $\cos(\arcsin x) = \cos(\theta) = \sqrt{1-x^2}$

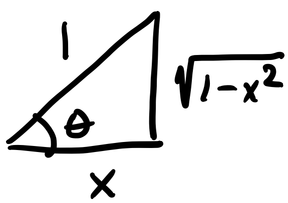
Similarly:

$y = \arccos x \iff x = \cos y$. So $\dfrac{dy}{dx} = \dfrac{-1}{\sin y} = \dfrac{-1}{\sin(\arccos x)}$

We can do better.

$= \dfrac{-1}{\sqrt{1-x^2}}$

Now



$\arccos x = \arccos\left(\frac{x}{1}\right) = \theta$

So $\sin(\arccos x) = \sqrt{1-x^2}$

Finally:

$y = \arctan x \iff x = \tan y$. So $\dfrac{dy}{dx} = \dfrac{1}{\sec^2 y} = \dfrac{1}{1+\tan^2 y}$

Pythagorean identity

$$= \frac{1}{1+(\tan(\arctan x))^2} = \frac{1}{1+x^2}$$

3. For your own reference, state the derivatives of $f(x) = \sin^{-1} x$, $f(x) = \cos^{-1} x$, and $f(x) = \tan^{-1} x$, in the space below:

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[\arctan x] = \frac{1}{1+x^2}$$

$$\frac{d}{dx}[\arccos x] = \frac{-1}{\sqrt{1-x^2}}$$

4. Find the derivatives of each of the following functions.

(a) $f(x) = \sin^{-1}(3x)$

$$f'(x) = \frac{1}{\sqrt{1-(3x)^2}} \cdot 3 = \frac{3}{\sqrt{1-9x^2}}$$

(b) $f(x) = (\cos^{-1} x)^2$

$$f'(x) = 2(\arccos x)^1 \cdot \frac{d}{dx}(\arccos x) = 2\arccos x \cdot \frac{-1}{\sqrt{1-x^2}} = \frac{-2\arccos x}{\sqrt{1-x^2}}$$

(c) $f(x) = x \tan^{-1} x$

$$f'(x) = 1 \cdot \arctan x + x \cdot \frac{1}{1+x^2} = \arctan x + \frac{x}{1+x^2}$$

(d) $f(x) = \arctan(\sqrt{4-x^2}) = \arctan\left((4-x^2)^{1/2}\right)$

$$f'(x) = \frac{1}{1+\left[(4-x^2)^{1/2}\right]^2} \cdot \frac{d}{dx}\left((4-x^2)^{1/2}\right) = \frac{1}{1+4-x^2} \cdot \frac{1}{2}(4-x^2)^{-1/2}(-2x) = \frac{-x}{(5-x^2)\sqrt{4-x^2}}$$

(e) $f(x) = \frac{\arcsin(\frac{1}{x})}{x} = x^{-1} \cdot \arcsin(x^{-1})$

$$f'(x) = -x^{-2} \cdot \arcsin(x^{-1}) + x^{-1} \cdot \frac{1}{\sqrt{1-x^{-2}}} \cdot -x^{-2} = \frac{-\arcsin(\frac{1}{x})}{x^2} - \frac{1}{x^3\sqrt{1-x^{-2}}}$$

$$= -\frac{1}{x^2}\left(\arcsin\left(\frac{1}{x}\right) + \frac{1}{x\sqrt{1-x^{-2}}}\right)$$