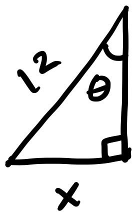
algebra aside $\frac{1}{\sqrt{1-\frac{x^2}{144}}} \cdot \frac{12}{12} = \frac{12}{\sqrt{144-x^2}}$

1. (Warm-up)A 12-foot ladder is leaning against a wall. Let x denote the distance of the base of the ladder from the wall, and let θ be the angle between the ladder and the wall.

   (a) How fast does the angle θ change with respect to x?



$\sin\theta = \frac{x}{12}$

We want $\frac{d\theta}{dt}$.

Solve for θ.

$\theta = \arcsin\left(\frac{x}{12}\right)$

$$\frac{d\theta}{dt} = \frac{1}{\sqrt{1-\left(\frac{x}{12}\right)^2}} \cdot \frac{1}{12} = \frac{1}{\sqrt{144-x^2}} \text{ rad/min}$$

Does this make sense?

   (b) I compute that $d\theta/dx \approx 0.1$ when $x = 7$. What does this mean in language your parents can understand? Feel free to express your answer in terms of degrees instead of radians.

$\frac{0.1 \text{ rad}}{} \cdot \frac{180^\circ}{\pi \text{ rad}} = \frac{18^\circ}{\pi} \approx \frac{18^\circ}{3} = 6^\circ$. So, parents, when the base of the ladder is 7 ft from the wall and being pushed away from the wall, the angle with the wall is increasing at a rate of about 6° per foot.

2. Vera says she is not a huge fan of logarithms so rewrites the function $\boxed{y = \ln x}$ as $\boxed{x = e^y}$. Is this ok?

Yes. This is a demonstration of the definition of $\ln x$ as inverse of e^x. (ie: How do you find the inverse of $y=f(x)$? Switch x & y and solve for y.)

3. Find $\frac{dy}{dx}$ implicitly for $x = e^y$ and write your answer in terms of x.

$1 = e^y \cdot \frac{dy}{dx}$. So $\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{e^{\ln x}} = \frac{1}{x}$

4. Find $\frac{dy}{dx}$ implicitly for $x = a^y$ and write your answer in terms of x.

$1 = (\ln a)\cdot a^y \cdot \frac{dy}{dx}$ So $\frac{dy}{dx} = \frac{1}{(\ln a)\cdot a^y} = \frac{1}{(\ln a)\, a^{\log_a x}} = \frac{1}{(\ln a)x}$

Congratulations, you just derived the formulas for the derivatives of logarithms. :)

Yay!

Using the formulas you just derived (and possibly the chain rule and/or the quotient rule and/or the product rule...) find the derivatives of each of the following:

5. $f(x) = (\ln x)^{7/2}$

(Chain rule)

$$f'(x) = \frac{7}{2}(\ln x)^{5/2} \cdot \frac{1}{x} = \frac{7(\ln x)^{5/2}}{2x}$$

6. $f(x) = \ln(\sqrt{x})$ $= \ln(x^{1/2}) = \frac{1}{2}\ln x$; So $f'(x) = \frac{1}{2}\cdot\frac{1}{x} = \frac{1}{2x}$

rules about logs

7. $f(x) = \ln(3x+1)$

(chain rule)

$$f'(x) = \frac{1}{3x+1} \cdot 3 = \frac{3}{3x+1}$$

8. Consider $y = \cancel{\left(\frac{x^2-2}{3-x}\right)}^3$ $\ln\left[\left(\frac{x^2-2}{3-x}\right)^3\right]$

(a) Without actually taking the derivative, list the rules you would need to do so.

- Chain rule
- quotient rule

(b) Use rules of logarithms, expand the right-hand side and then take the derivative.

$$y = \ln\left(\left(\frac{x^2-2}{3-x}\right)^3\right) = 3\ln\left(\frac{x^2-2}{3-x}\right) = 3\left[\ln(x^2-2) - \ln(3-x)\right]$$

$$\frac{dy}{dx} = 3\left[\frac{2x}{x^2-2} - \frac{-1}{3-x}\right] = \frac{6x}{x^2-2} + \frac{3}{3-x}$$

#8 can be exploited:

$y = (\cos x)^x$ why don't any previous rules work?

New problem

$\ln y = \ln\left[(\cos x)^x\right]$

$\ln y = x \ln(\cos x)$

take derivative implicitly

$\frac{1}{y} \cdot y' = 1 \cdot \ln(\cos x) + x \cdot \frac{1}{\cos x} \cdot -\sin x$

$\frac{dy}{dx} = y\left[\ln(\cos x) - x\tan x\right]$

$= (\cos x)^x \left[\ln(\cos x) - x\tan x\right]$

Sketch $y = \ln|x|$.

Can you give a formula for just the LHS?

$f(x) = \ln(-x)$

$f'(x) = \frac{1}{-x} \cdot -1 = \frac{1}{x}$