§3.9 Start up

- implicit differentiation snap shot

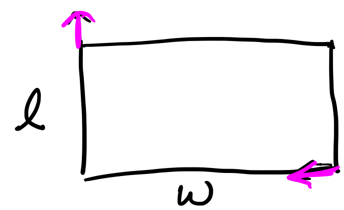
assume y an implicit fcn of x

$$\boxed{\sin y}$$ ← something complicated

$$(\cos y)\cdot\frac{dy}{dx}$$

- What if we have multiple variables (things) changing over time?

- Example



- l is getting longer at a rate of 3in/min
- w is getting shorter at a rate of 2in/min

How is area changing when $l=4$ and $w=7$?

$$A = l\cdot w$$

$$\frac{dA}{dt} = l\cdot\frac{dw}{dt} + w\cdot\frac{dl}{dt}$$

when the instant, $\frac{dA}{dt} = 4\cdot(-2) + 7\cdot 3 = -8+21 = 13 \text{ in}^2/\text{min}$

1. A pebble dropped into a calm pond, causing ripples in the form of circles. The radius r of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the area A of the water disturbed changing?

We want $\frac{dA}{dt}$ when $r=4$. We know $\frac{dr}{dt}=1.\ ft/s$.

$A=\pi r^2$

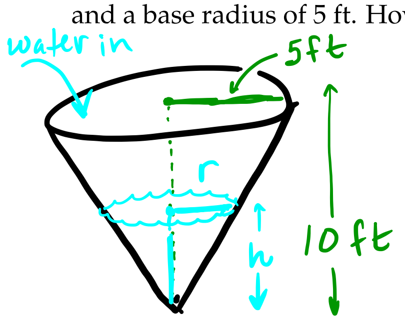
$\frac{dA}{dt}=2\pi r\frac{dr}{dt}$

When $r=4$,

$\frac{dA}{dt}=2\pi\cdot 4\cdot 1=8\pi\ ft^2/s$

The area is increasing at a rate of $8\pi\approx 25\ ft^2/s$ when the radius is 4ft.

2. Water runs into a conical tank at the rate of 9 ft^3/min. The tank stands point down and has a height of 10 ft and a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep?



$\frac{dV}{dt}=9\ ft^3/min$

We want $\frac{dh}{dt}$ when $h=6$ ft.

We need an equation relating h and V for a cone.

We know $V=\frac{1}{3}\pi r^2h$.

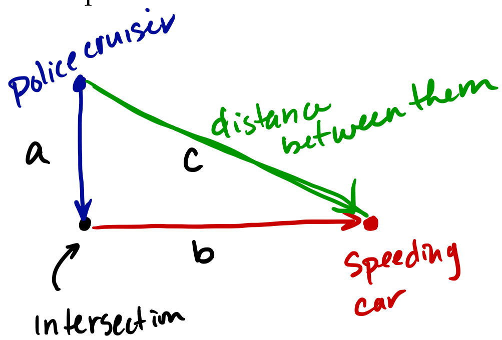
Can we eliminate r? Using similar triangles: $\frac{r}{h}=\frac{5}{10}$ or $\boxed{r=\frac{1}{2}h}$.

Substitute: $V=\frac{1}{3}\pi\left(\frac{1}{2}h\right)^2h=\frac{\pi}{12}h^3$. So $\frac{dV}{dh}=\frac{\pi}{4}h^2\cdot\frac{dh}{dt}$

At $h=6$, $9=\frac{\pi}{4}\cdot 6^2\cdot\frac{dh}{dt}$. Or $\frac{dh}{dt}=\frac{1}{\pi}\approx 0.318\ ft/min$.

When the height of water is 6 ft, the height is increasing at a rate of about 0.318 ft/min.

3. A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 mi north of the intersection and the car is 0.8 mi to the east, the police determine that the distance between them and the car they are chasing is increasing at a rate of 20 mph. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car?



$a^2 + b^2 = c^2$

When $a = 0.6$ mi, $b = 0.8$ mi, and $\frac{dc}{dt} = 20$ mph,

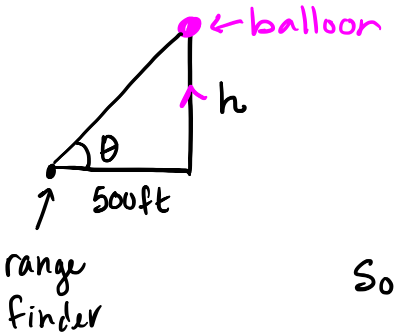
and $\frac{da}{dt} = -60$ mph

We want $\frac{db}{dt}$.

$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$. So $\frac{db}{dt} = \left[c\frac{dc}{dt} - a\frac{da}{dt}\right] / b$. (We need c: $c^2 = (.8)^2 + (.6)^2$ so $c = 1$)

So $\frac{db}{dt} = \left[1 \cdot 20 - (0.6)(-60)\right] / 0.8 = 70$ mph. At the moment in this scenario, the car is traveling at 70 mph away from the intersection.

4. A hot air balloon rising straight up from a level field is tracked by a range finder 500 feet from the lift-off point. At the moment the range finder's elevation angle is $\pi/4$, the angle is increasing at the rate of 0.14 radians/min. How fast is the balloon rising at that moment?

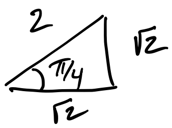


We want $\frac{dh}{dt}$ when $\theta = \frac{\pi}{4}$ and $\frac{d\theta}{dt} = 0.14$

So $\tan\theta = \frac{h}{500}$ or $h = 500\tan\theta$.

So $\frac{dh}{dt} = 500\sec^2\theta \cdot \frac{d\theta}{dt}$. When $\theta = \frac{\pi}{4}$ and $\frac{d\theta}{dt} = 0.14$,

$$\frac{dh}{dt} = 500 \cdot \sec^2\left(\frac{\pi}{4}\right) \cdot (0.14) = 500 \cdot 2 \cdot 0.14 = 140 \text{ ft/min}$$



$\sec\frac{\pi}{4} = \frac{2}{\sqrt{2}} = \sqrt{2}$

$\sec^2\pi/4 = 2$

So at the moment the range finder is at an angle of 45°, the balloon is rising at a rate of 140 ft/min.