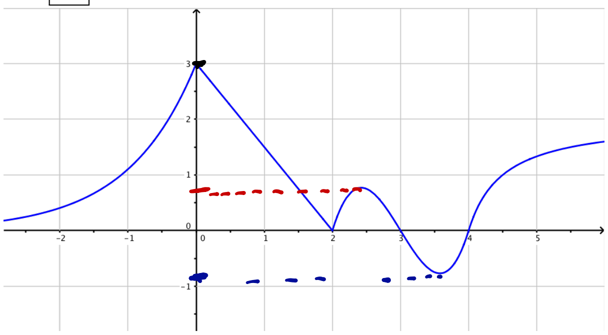
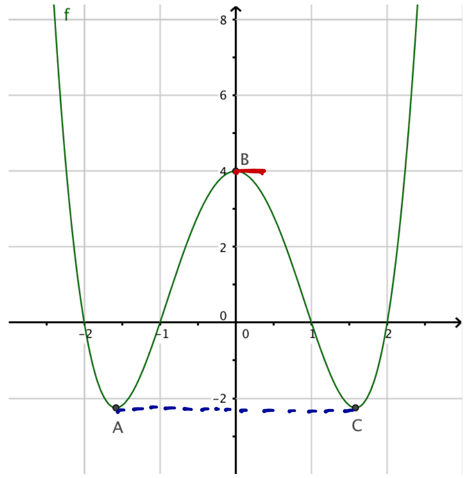
Lecture Notes: 4-1 Maximum and Minimum Values

Motivating Examples

A



B



absolute max = 3

absolute min = -0.9 at $x = 3.5$

local max = 0.75 at $x = 2.5$

local min = 0 at $x = 2$

(also -0.9 mentioned above.)

absolute max = none

absolute min = -2.3 at $x = -1.5$ and $x = 1.5$

local max = 4 at $x = 0$

local min: none other than abs. min above

Definitions: Let $f(x)$ be a function with domain D and let c be an x-value in D. Then the y-value $f(c)$ is:

1. **an absolute maximum** if $f(c) \geq f(x)$ for all x in D.
2. **an absolute minimum** if $f(c) \leq f(x)$ for all x in D.
3. **a local maximum** if $f(c) \geq f(x)$ for all x close to c.
4. **a local minimum** if $f(c) \leq f(x)$ for all x close to c

Are we all on the same page?

1. What sort of *category* is a maximum (or minimum)? (Animal, vegetable, number, point, x-value, y-value, mineral...?)

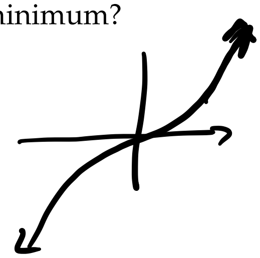
max's and min's are y-values or function outputs
or real numbers.

2. Can function have more than ONE maximum (or minimum)?

absolute max? only one. May occur at many places.
local max? more than one.
(same for mins)

3. Can a function have neither a maximum nor a minimum?

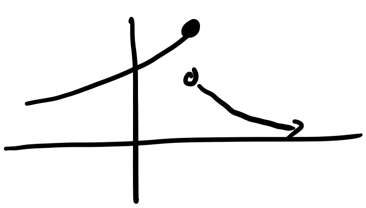
Sure. $y=x^3$ is an example.



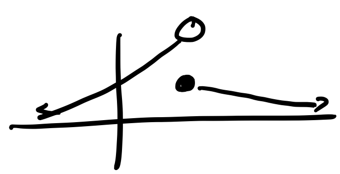
4. Looking at our earlier pictures, at what sort of places do maximums and minimums appear?

where $f'=0$ or where f' is undefined.

5. What happens if the graph is not continuous? Draw some pictures.



or



Definition: A **critical number** of a function $f(x)$ is an $\underline{\;x\;}$-value c $\underline{\text{ in the domain of } f(x)}$ such that either

(a) $f'(c)=0$ *or* (b) $f'(c)$ is undefined.

Without using your calculator, for each function below:

a) Sketch the graph.

b) Find any critical points or explain why none exist.

c) Identify any absolute and local maximum and maximum values of f and state where they occur. If none exist, state this explicitly.

1. $f(x) = 5 + 54x - 2x^3$

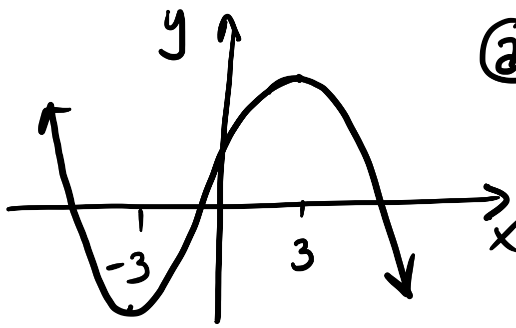
$f'(x) = 54 - 6x^2 = 6(9-x^2) = 6(3-x)(3+x)$

$f'(x) = 0$ when $x = \pm 3$ ← answer to (b)

f' is never undefined

(c) no absolute max or min.

(a)

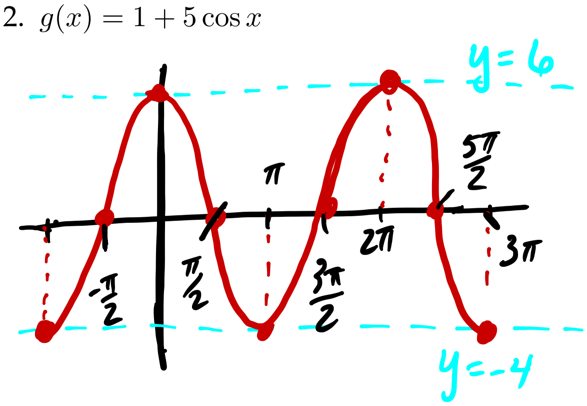


local max : $f(3) = 113$ at $x=3$

local min : $f(-3) = -103$ at $x=-3$

2. $g(x) = 1 + 5\cos x$

a.



b. $x = k\pi$, k integer

crit. pts.

[from graph OR $g'(x)$]

$g' = -5\sin x = 0$ when $\sin x = 0$

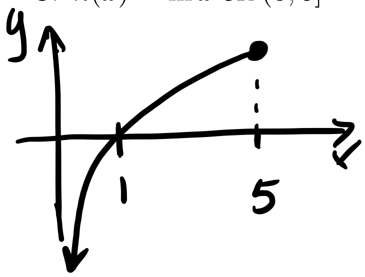
or $x = k\pi$.

absolute max: $y=6$ at $x = 2\pi k$,

absolute min: $y=-4$ at $x = (2k+1)\pi$

no local min or max other than the absolute ones.

3. $h(x) = \ln x$ on $(0, 5]$



(a)

(c) no rel min or absolute min

absolute max: $y = \ln 5$ at $x = 5$

The absolute max is the only local max

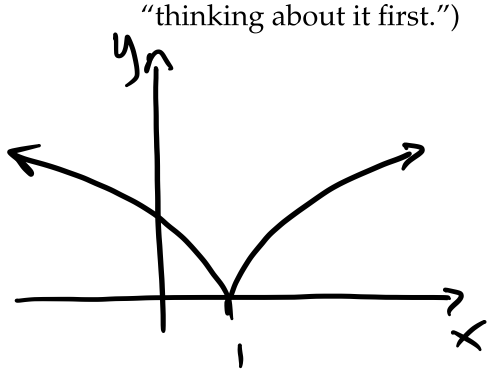
(b) no crit. pts.

from graph OR

$h'(x) = \frac{1}{x}$ and

$h' \neq 0$ and h' undefined at $x = 0$ but it's not in the domain.

4. $f(x) = (x-1)^{2/3}$ (This one you may graph on a calculator *after* you have tried the technique called "thinking about it first.")



(b) $f'(x) = \frac{2}{3}(x-1)^{-1/3} = \frac{2}{3(x-1)^{1/3}}$

f' undefined at $x=1$.

f' never 0.

c.pts: $x = 1$

(c) absolute min: $y = 0$ at $x = 1$.

no absolute or local max.

local min only: $y = 0$